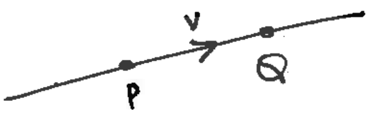
MATH 2850
Quiz 1
September 9, 2016

NAME. SOLUTION

Let $Q = (1,1,-1)$ and $P = (-1,2,0)$ be given two points in $\mathbb{R}^3$.

1. a. Find a parametric representation for the line ℓ passing through Q and P.



$$v = Q - P = (1,1,-1) - (-1,2,0) = (2,-1,-1)$$

$$r(t) = P + tv = (1,1,-1) + t(2,-1,-1)$$
$$= (1+2t,\ 1-t,\ -1-t)$$

OR $\begin{cases} x = 1+2t \\ y = 1-t \\ z = -1-t \end{cases}$

b. Find the point where the line ℓ (form part **a**) intersects the plane $x - y + z = 3$.

Solve

$$x = 1+2t$$
$$y = 1-t$$
$$z = -1-t$$

$$x - y + z = 3$$
$$(1+2t) - (1-t) + (-1-t) = 3$$
$$\cancel{1} + 2t - \cancel{1} + \cancel{t} - 1 - \cancel{t} = 3$$
$$2t - 1 = 3 \implies t = 2$$

$$t = 2$$
$$\implies x = 1 + 2\cdot 2 = 5$$
$$y = 1 - 2 = -1$$
$$z = -1 - 2 = -3$$

pt of intersection $(5, -1, -3)$.

2. a. Calculate $proj_{\mathbf{a}}\mathbf{b}$ where $\mathbf{a} = \mathbf{i} + \mathbf{k}$ and $\mathbf{b} = \mathbf{i} + \mathbf{j}$.

$$proj_{i+k}(i+j) = \frac{(i+j)\cdot(i+k)}{(i+k)\cdot(i+k)}(i+k) = \frac{1+0+0}{1+0+1}(i+k)$$

$$= \frac{1}{2}(i+k)$$

b. Find all values of $a \in \mathbb{R}$, if the angle between the vectors $\underbrace{a\mathbf{i} + \mathbf{j} + \mathbf{k}}_{v_1}$ and $\underbrace{\mathbf{i} + \mathbf{j} + a\mathbf{k}}_{v_2}$ is $60°$.

$$v_1 \cdot v_2 = \|v_1\| \cdot \|v_2\| \cos\theta$$

$$v_1 = ai + j + k \implies \|v_1\| = \sqrt{a^2+1+1} = \sqrt{a^2+2}$$

$$v_2 = i + j + ak \implies \|v_2\| = \sqrt{1+1+a^2} = \sqrt{a^2+2}$$

$\cos\theta$

$$v_1 \cdot v_2 = a + 1 + a = 2a + 1 = \sqrt{a^2+2}\,\sqrt{a^2+2}\cdot\frac{1}{2} = \frac{1}{2}(a^2+2)$$

$$2a + 1 = \frac{1}{2}(a^2+2) \implies 4a + \cancel{2} = a^2 + \cancel{2}$$
$$\implies 4a = a^2$$

So: $\boxed{a = 0 \text{ OR } a = 4.}$