MATH 2850
Optional QUIZ
Dec. 8, 2016

NAME. SOLUTION

8 pts

1. Calculate $\int_C \vec{F} \cdot \vec{ds}$, where $\vec{F} = z\vec{i} + 2y\vec{j} + x\vec{k}$, and C is the curve parametrized by $\vec{x}(t) = (t, 3t^2, 2t^3)$, $0 \le t \le 2$.

$$\vec{x}'(t) = (1, 6t, 6t^2)$$

$$F(\vec{x}(t)) = (2t^3, 6t^2, t)$$

$$\int_C \vec{F} \cdot d\vec{s} = \int_0^2 (2t^3, 6t^2, t) \cdot (1, 6t, 6t^2)\, dt$$

$$= \int_0^2 (2t^3 + 36t^3 + 6t^3)\, dt = \int_0^2 44t^3\, dt = 11t^4 \Big|_0^2$$

$$= 11 \cdot 16 = 176$$

12 pts

2. Let L is the line segment from $(1,0)$, to $(3,-1)$ in the xy-plane.

(4) i. Find a parametrization of L.

(4) ii. Calculate $\int_L y^2 dx + x^2 dy$

(4) iii. Calculate $\int_L xy\, ds$

$$\vec{x}(t) = (1-t)(1,0) + t(3,-1) \qquad 0 \le t \le 1$$

$$= (1-t, 0) + (3t, -t)$$

(i) $\boxed{\vec{x}(t) = (1+2t, -t)} \longrightarrow \vec{x}' = (2, -1)$

(ii) $$\int y^2 dx + x^2 dy = \int_0^1 (-t)^2 \cdot 2dt + (1+2t)^2 \cdot (-dt)$$

$$\boxed{\begin{matrix} x = 1+2t & dx = 2dt \\ y = -t & dy = -dt \end{matrix}}$$

$$= \int_0^1 (+2t^2 - 1 - 4t - 4t^2)\, dt = \int_0^1 (-2t^2 - 4t - 1)\, dt$$

$$= -\frac{2}{3}t^3 - 2t^2 - t \Big|_0^1 = -\frac{2}{3} - 2 - 1 = \boxed{-\frac{11}{3}} \text{ (ii)}$$

(iii) $$\int xy\, ds = \int_0^1 (1+2t) \cdot (-t) \sqrt{5}\, dt = \int_0^1 \sqrt{5}(-t - 2t^2)\, dt$$

$$ds = \|\vec{x}'(t)\|\, dt$$

$$= \sqrt{5}\left(-\frac{t^2}{2} - \frac{2t^3}{3}\right)\Big|_0^1 = \sqrt{5}\left(-\frac{1}{2} - \frac{2}{3}\right) = \frac{-7\sqrt{5}}{6}.$$

$$ds = \|(2, -1)\|\, dt$$

$$ds = \sqrt{5}\, dt$$