

General Comments and Recommendations for Learning How to Write a Proof

1. Before reading or writing a proof, review the relevant definitions, axioms and the statements of the theorems already established.
2. About reading a proof of a particular theorem written by someone else:
 - Be critical of each statement. In order to convince yourself that something is true, you must ask “why” repeatedly. Any reasoning is as important as the statement itself, if not more.
 - Think about why a particular statement was put in the proof, and what can follow that statement.
 - After checking each statement, see how everything is put together, figure out which step depends on which other steps, and understand why the author chose a certain logical path to put them together. Did the author start with the given/hypothesis, and was the author able to connect to the conclusion correctly? Are you convinced?
 - Compare the last proof you read to some of the previous ones. Are there similar ones or contrasting ones or independent?
 - Do not assume a proof is absolutely correct if it is in print in a book, online, etc. There are lots of wrong proofs around.
 - Can you repeat the same proof by writing on a blank piece of paper?
3. Reading proofs by others is easier than writing proofs, especially for the beginners. However, you need to learn how to write your own proofs. Otherwise, you end up memorizing other people’s work (quite boring), but not develop your own logical and writing skills which you will need in the future.
4. There are many different types of proofs, but the following rules are common to all types when you are writing a proof.
 - a. You must use complete sentences, and complete mathematical statements if symbolic notation is used.
 - b. It is a very good idea to identify your starting point and write it down in the beginning of the proof. This is a necessity in this course.
 - i. DO NOT START WITH THE OPPOSITE OF THE HYPOTHESIS. NEVER!
 - ii. DO NOT WRITE THE FINAL CONCLUSION OF YOUR PROOF AS A STATEMENT IN THE BEGINNING: read c and d.

- c. Every statement you write down must be mathematically correct and justified immediately or by the previous lines of the same proof, or earlier results which are already proven, or axioms and definitions. No statement can be justified by using other statements appearing later in the proof. This is for avoiding circular logic. It is common to justify a statement, by giving a justification immediately after, without separation and before going to the next step, such as in the following ways:
- i. P is true. For,.....
 - ii. P is true (this is why...)
 - iii. P is true, otherwise....
 - iv. Claim 1: P is true. Proof the Claim 1: (written as a sub-proof within the main proof, but sub-proof must be finished before going any further in the main proof.)
- d. You do NOT write your final conclusion in the beginning of a proof as a statement. Why? In a proof, what is written down is claimed to be true by you at that time. If you have not given the proof and sufficient reasoning yet, you will be making a claim without basis. People will say your proof is not correct/convincing. However, it is fine to write "This is what I want to prove:", and not pretending that you finished the proof before you started. "This is what I want to prove:" is not part of your formal proof, but it is a comment to make the reader's life easier.
- e. There must be logical continuity and order in your writing. It is not the readers job to rearrange your claims to make some sense out of them. Usually, rearranging the statements of a correct proof will produce a wrong proof. Hence, the order of the statements is very important.
- f. There cannot be gaps of logical continuity between your claims. A bridge does not connect two sides if the middle of the bridge is missing. A proof is false if parts are missing. An incomplete proof (with gaps) is no proof.
- g. Read what you have written down. Be EXTREMELY CRITICAL of your own writing, before you let someone else read it with your name on it. Ask yourself repeatedly why you put a statement in your proof, whether it is correct and justified. Check everything again. If you cannot convince yourself, or you do not believe in or understand what you wrote, then it will show and no one else will believe it.