Announcements

① HW #5 is due Sept 29, 2017; but it will not be considered late if returned on Monday Oct 2.

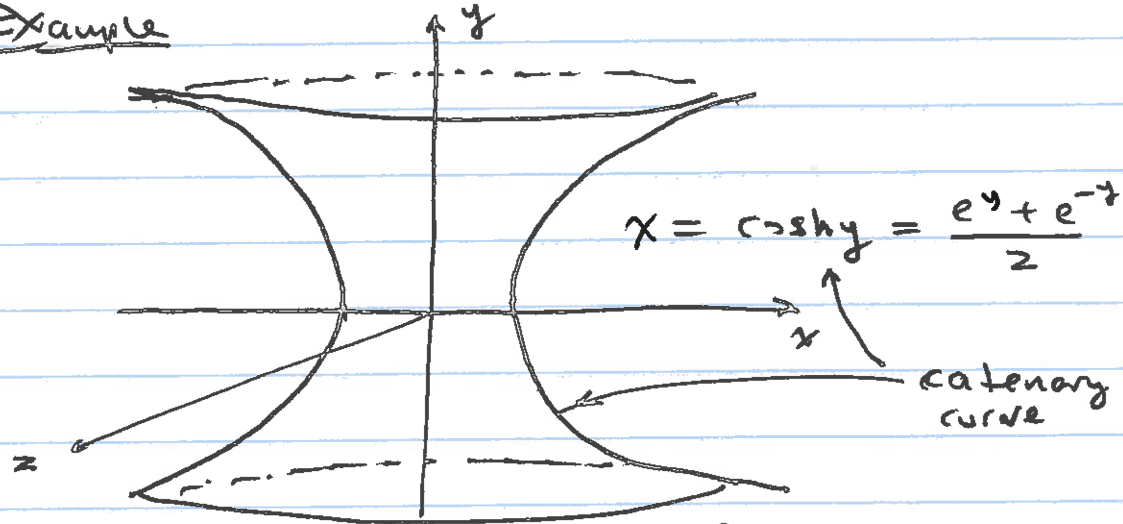
② MT will be handed out on Sept 29, Friday. MT is due Oct 9, Monday.

③ You are allowed to ask questions about the MT in class on Oct 4, Oct 6. The hints will be given to the whole class.

* Do not wait until Oct 7, 8 to start this MT exam.

④ No HW due oct 6.

Example

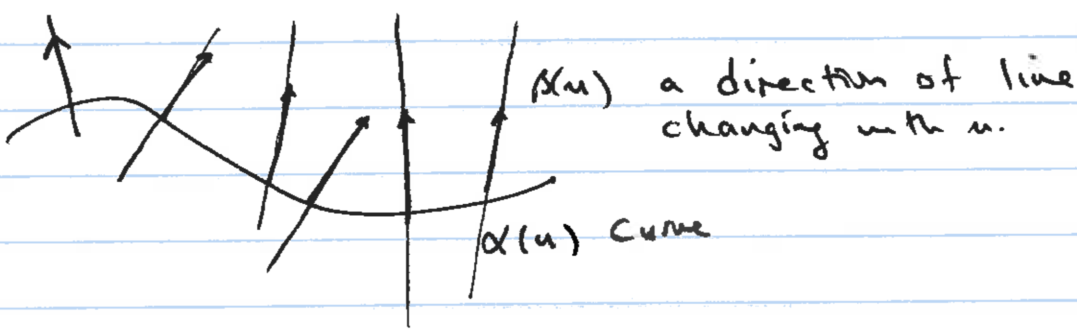


Rotate about y-axis.

catenoid

Minimal Surface, "locally minimizes area"

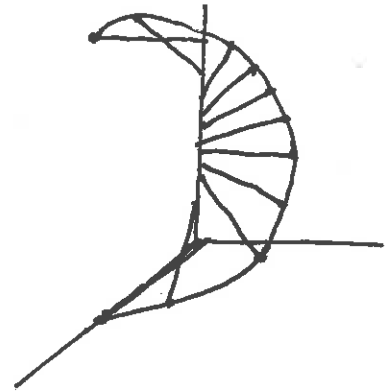
Ruled Surfaces



$$\varphi(u,v) = \alpha(u) + v\beta(u)$$

Special
$\begin{cases} \alpha(u) = \alpha_0 \text{ constant} & \text{Cones} \\ \beta(u) = \beta_0 \text{ constant} & \text{Cylinders} \end{cases}$

2



Also see the book p 7[illegible]

Ex 2.1.14 Helicoid

$$\Psi(u,v) = (0,0,u) + v(\cos u, \sin u, 0)$$

$$\Psi_u(u,v) = (0,0,1) + v(-\sin u, \cos u, 0)$$
$$= (-v\sin u, v\cos u, 1)$$

$$\Psi_v(u,v) = (\cos u, \sin u, 0)$$

(2.1.15) $N = \Psi_u \times \Psi_v = (-\sin u, \cos u, -v) \neq \vec{0}$

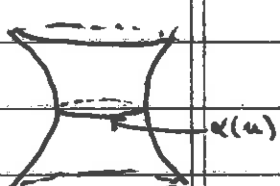
can't be 0 simultaneously

Ex A Ruled surface along: $x^2+y^2-z^2=1$, Hyperboloid of one sheet

2.1.22

$\boxed{\text{gives a surface of revolution parametrization}}$

$$\alpha(u) = (\cos u, \sin u, 0)$$
$$\beta(u) = (-\sin u, \cos u, 1)$$



$$\Psi(u,v) = \alpha(u) + v\beta(u)$$
$$= (\underbrace{\cos u - v\sin u}_{x}, \underbrace{\sin u + v\cos u}_{y}, \underbrace{v}_{z})$$

$\beta(u) = T(u) + (0,0,1)$

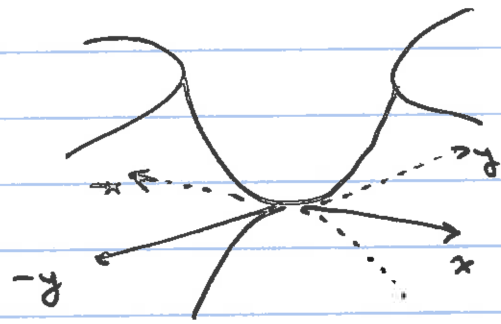
$T = (-\sin u, \cos u, 0)$

$\alpha(u) = (\cos u, \sin u, 0)$

This Ruled Surface is actually along hyperboloid of one sheet

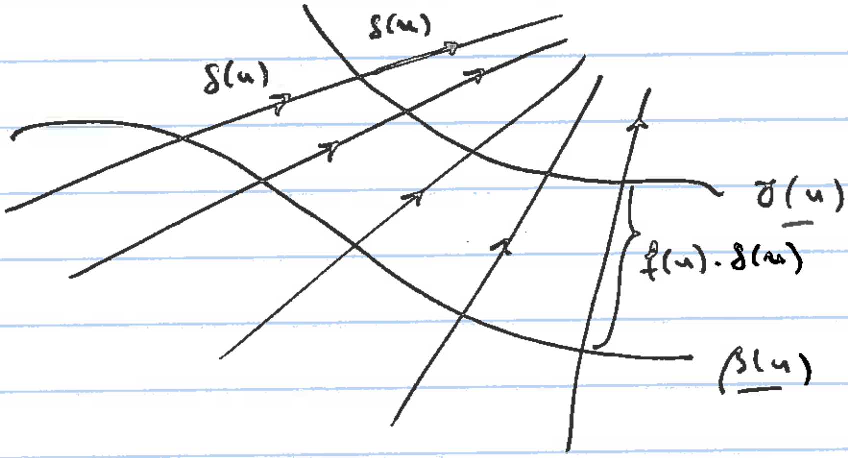
$$x^2+y^2-z^2 = \cos^2 u - \cancel{2v\cos u \sin u} + v^2\sin^2 u +$$
$$\sin^2 u + \cancel{2v\sin u\cos u} + v^2\cos^2 u - v^2$$
$$= 1 + v^2 - v^2 = 1.$$

Ex $f(x,y) = 2xy = z.$ $\longrightarrow$ observe that x-axis & y-axis lie on this graph.



$g(x,y) = x^2 - y^2 = z$ (Almost same graph but $45°$ rotated)

Hint 2.1.24



$|f(u)|$ length
$\delta(u)$ direction

$$\Phi(u,v) = \beta(u) + v\,\delta(u)$$
$$\Psi(u,w) = \gamma(u) + w\,\delta(u)$$

parametrize same surface, if we obtain $\gamma(u)$ by sliding $\beta(u)$ along the lines of the ruled surface:

$$\vec{\gamma}(u) = \vec{\beta}(u) + f(u)\cdot\vec{\delta}(u)$$

To find when you want $\gamma(u)$ to be a line of striction.