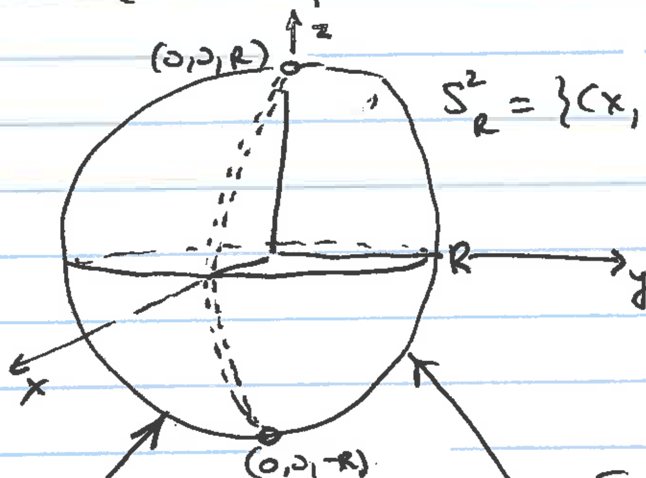
Ex. Sphere (other parametrizations)



$S^2_R = \{(x,y,z) \mid x^2+y^2+z^2 = R^2\}$

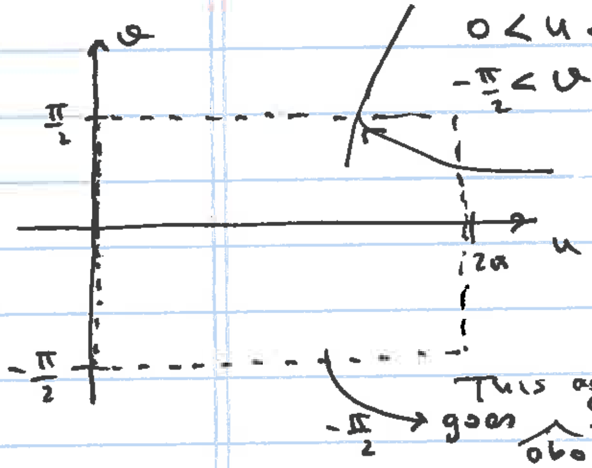
Geographical Ψ

Spherical coordinate parametriz. Φ

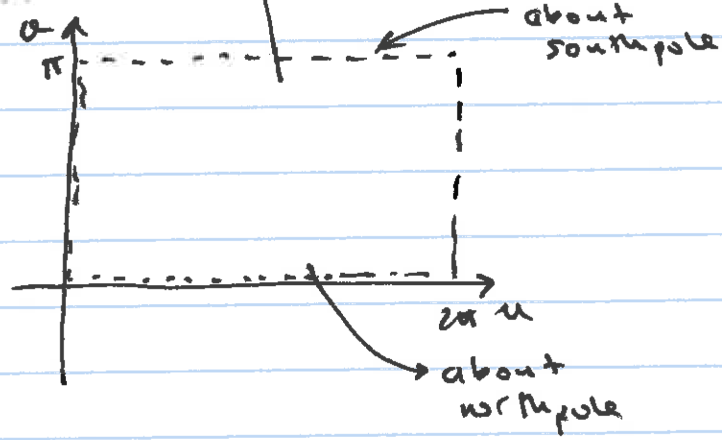
$\Psi(u,v) = R(\cos u \cos v, \sin u \sin v, \sin v)$

$0 < u < 2\pi$

$-\frac{\pi}{2} < v < \frac{\pi}{2}$



$\Phi(u,v) = R(\cos u \sin v, \sin u \sin v, \cos v)$



Calculate N for Φ

$$\begin{array}{ccc} i & j & k \end{array}$$

$$\Phi_u = R(-\sin u \sin v, \cos u \sin v, 0)$$

$$\Phi_v = R(\cos u \cos v, \sin u \cos v, -\sin v)$$

$$\Phi_u \times \Phi_v = N = (-\sin^2 v \cos u, -\sin^2 v \sin u, -\sin v \cos v) \cdot R^2$$

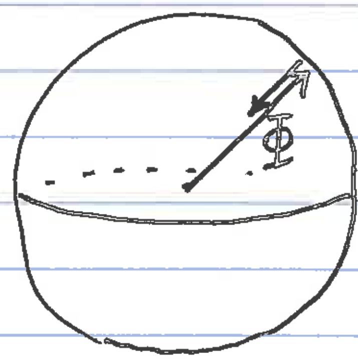
simplify

$$-\sin^2 u \sin v \cos v - \cos^2 u \sin v \cos v$$

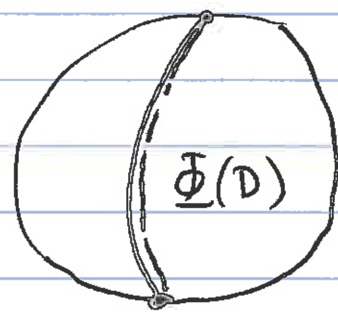
$$N = -R^2 \sin v \, \Phi$$

$N \neq 0$ if $\sin v \neq 0$

$v \neq 0, \pi$



Φ covers S^2 minus the slit between $(0,0,R)$, and $(0,0,-R)$ along xz plane, $x \geq 0$

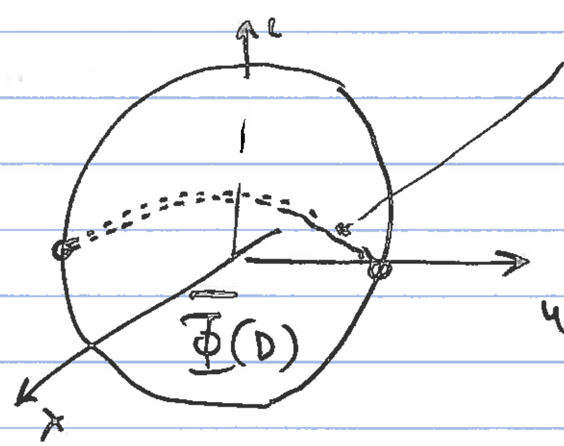


$$D = \left\{ (u,v) \,\middle|\, \begin{array}{l} 0 < u < 2\pi \\ 0 < v < \pi \end{array} \right\}$$

$$\bar{\Phi} = R(-\cos u \sin v,\ \cos v,\ \sin u \sin v)$$

$$\Phi = R(\cos u \sin v,\ \sin u \sin v,\ \cos v)$$

$\bar{\Phi}$ covers

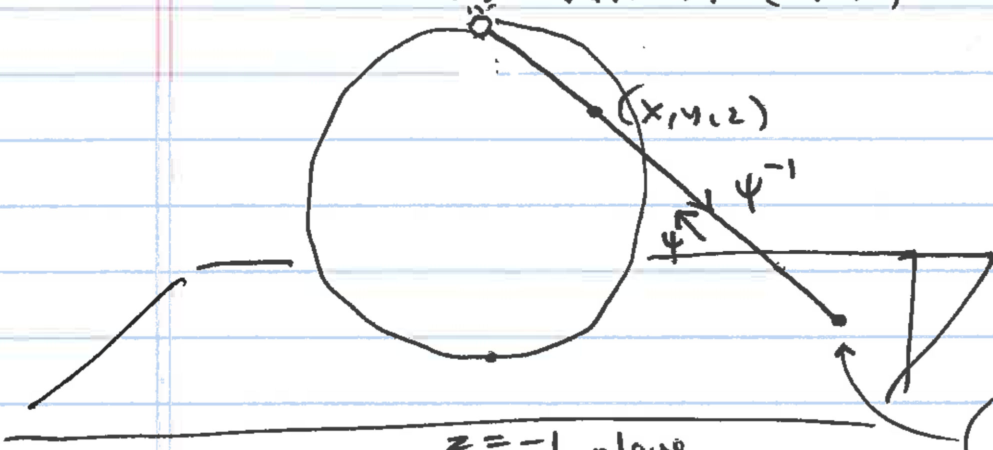


missing part is a slit between $(0,R,0)$ and $(0,-R,0)$ along xy-plane, $x \leq 0$.

$$\Phi(D) \cup \bar{\Phi}(D) = S^2_R = \{(x,y,z) \,|\, x^2+y^2+z^2=R^2\}$$

Stereographic projection

light bulb at $(0,0,1)$ north pole



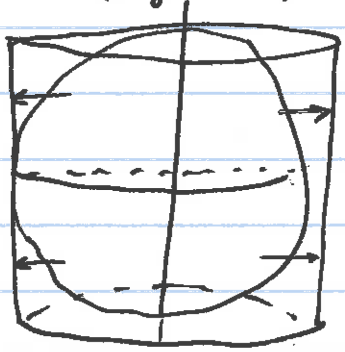
$$\left(\frac{x}{1-z}, \frac{y}{1-z}\right) = \psi^{-1}(x,y,z)$$

$$\psi(u,v) = \frac{1}{1+u^2+v^2}\left(2u, 2v, u^2+v^2-1\right)$$

- Stereographic projection preserves angles (Conformal)

Lambert's projection preserves area.

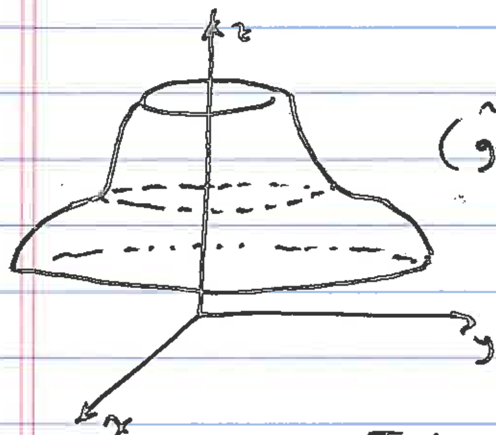
long light bulb thru $(0,0,1)$, $(0,0,-1)$



project side ways to a cylinder
then cut the cylinder along a vertical line
and open flat.
(unlike sphere, a cut cylinder can open flat without ripping)

(*) No projection of S^2 on $\mathbb{R}^2$ preserves distance / lengths of curves
(will prove later) (any (piece of) sphere cannot be flattened without ripping.)

Surfaces of Revolutions



$\left(\overset{y}{g(u)}, \overset{z}{h(u)}\right)$ curve in yz plane $u \in I \subseteq \mathbb{R}^1$
rotate about z-axis.

same since rotated about z-axis.

$$\Phi(u,v) = \left(g(u)\cos v,\ g(u)\sin v,\ h(u) \right)$$

$u \in I$ and each $\to$ $0 < v < \pi$ gives one chart
each $\to$ $\pi - \varepsilon < v < 2\pi + \varepsilon$ gives another chart.

2.1.
Exc#11 Calculate N for Φ above (rotation about z-axis)

$$\Phi_u = \left(g'(u)\cos v,\ g'(u)\sin v,\ h'(u) \right)$$

$$\Phi_v = \left(-g(u)\sin v,\ g(u)\cos v,\ 0 \right)$$

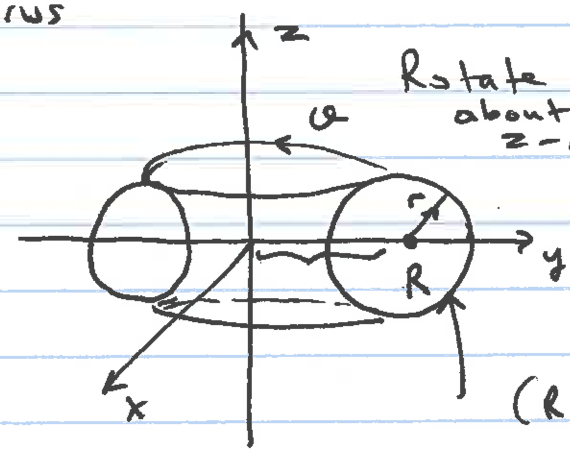
$$\Phi_u \times \Phi_v = N = \left(-h'(u)g(u)\cos v,\ -g(u)h'(u)\sin v,\ g(u)g'(u) \right)$$

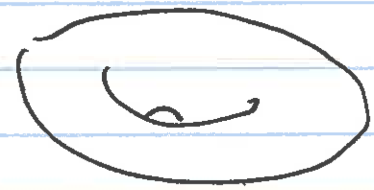
$$= g(u)\left(-h'(u)\cos v,\ -h'(u)\sin v,\ g'(u) \right)$$

Caution: Compare to book, where the rotation is about x-axis p72,73

Sept 22, 2017

Exc #13 Torus





$$(R,0) + r(\cos u, \sin u)$$
$$= (R + r\cos u,\ \underbrace{r \sin u}) = (y, z)$$

$$\big((R + r\cos u)\cos v,\ (R + r\cos u)\sin v,\ r \sin u \big)$$

$$\left.\begin{matrix} 0 < u < \pi \\ 0 < v < \pi \end{matrix}\right] \qquad \left.\begin{matrix} \pi - \varepsilon < u < 2\pi + \varepsilon \\ 0 < v < \pi \end{matrix}\right]$$

$$\left.\begin{matrix} 0 < u < \pi \\ \pi - \varepsilon < v < \pi + \varepsilon \end{matrix}\right] \qquad \left.\begin{matrix} \pi - \varepsilon < u < 2\pi + \varepsilon \\ \pi - \varepsilon < v < 2\pi + \varepsilon \end{matrix}\right]$$

use 4 different domains to cover Torus by regular, 1-1 parametrizations.