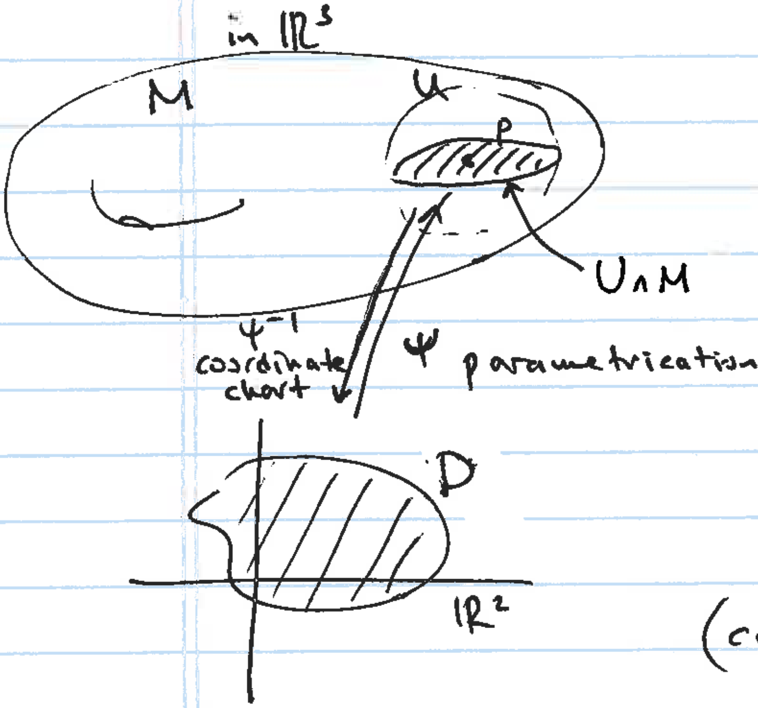
2.1

Defn A subset $M \subseteq \mathbb{R}^3$ is called a (regular) surface if

$\forall p \in M \ \exists U^{open} \subseteq \mathbb{R}^3$

$\exists D^{open} \subseteq \mathbb{R}^2$

$\exists$ a regular parametrization



$\psi : D \longrightarrow U \cap M$ which is a homeomorphism

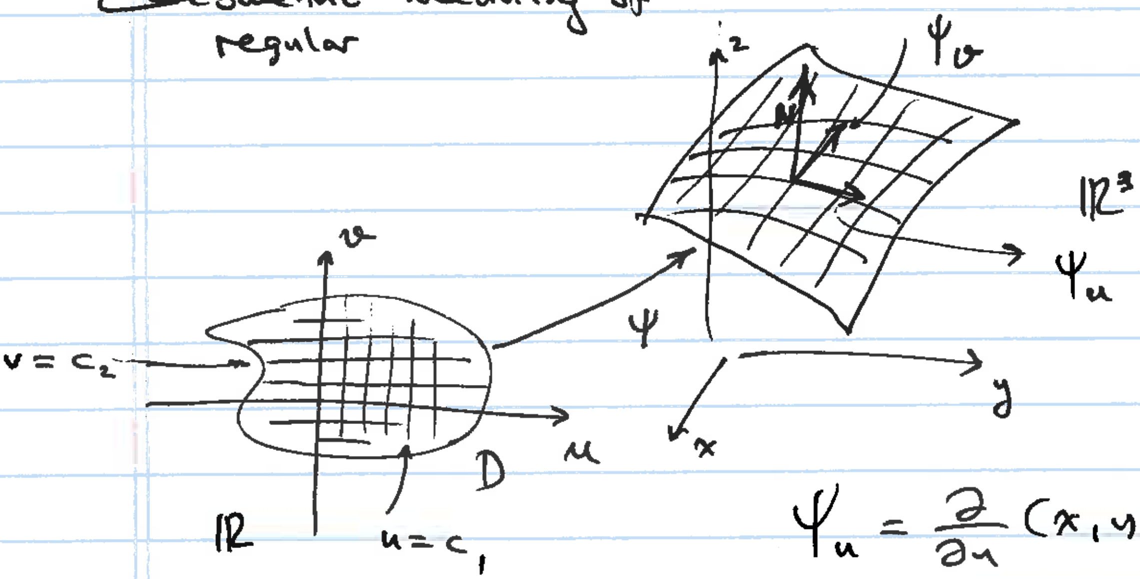
that is:

ψ 1-1, onto, continuous

$\psi^{-1} : U \cap M \longrightarrow D$ continuous

(called coordinate chart)

Geometric Meaning of regular

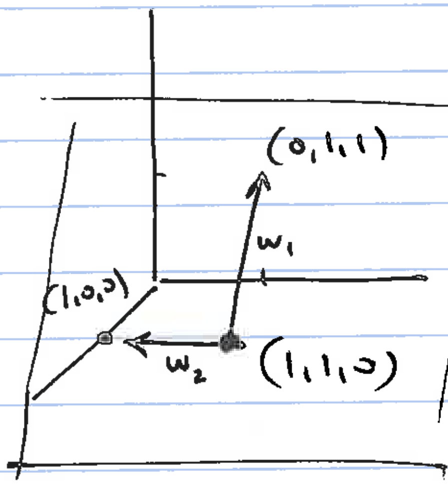


$$\psi_u = \frac{\partial}{\partial u}(x, y, z) = \frac{\partial \psi}{\partial u}$$

$$\psi_v = \frac{\partial}{\partial v}(x, y, z) = \frac{\partial \psi}{\partial v}$$

$$\vec{0} \neq N = \psi_u \times \psi_v$$

Ex1 $\varphi(u,v) = (1,1,0) + u\underbrace{(-1,0,1)}_{w_1} + v\underbrace{(0,-1,0)}_{w_2}$



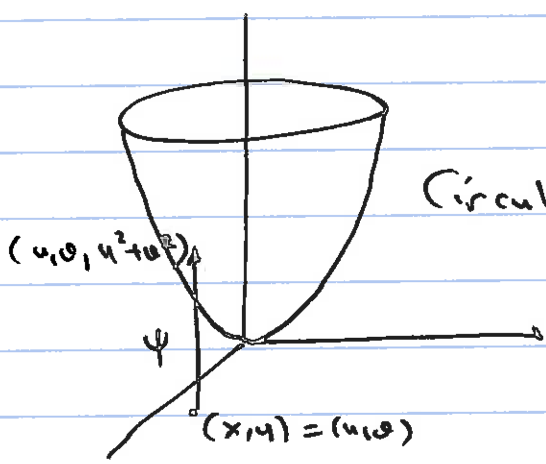
parametric 2 plane in $\mathbb{R}^3$

thru $(1,1,0)$

// $(-1,0,1)$

// $(0,-1,0)$

Ex2 (a) $\varphi(u,v) = (u, v, u^2+v^2)$

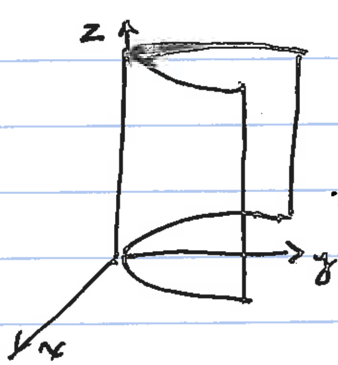


(Monge Parametrization of a graph.)

Circular Paraboloid

$$z = x^2 + y^2 = f(x,y)$$

(b) $\Phi(u,v) = (u, u^2, v)$



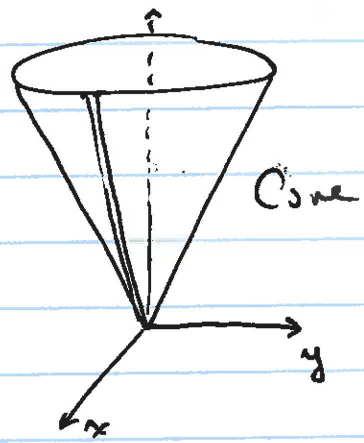
$$g(x,z) = y = x^2$$

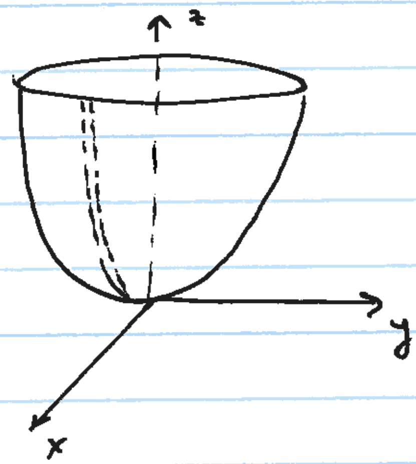
(Monge parametrization of this graph.)

Parabolic Cylinder

Ex 3 $\quad \Psi_1(r,\theta) = (r\cos\theta, r\sin\theta, r^2)$ $\quad\quad 0<\theta<2\pi$

$\Psi_2(r,\theta) = (r\cos\theta, r\sin\theta, r)$ $\quad\quad 0<r$





$$(\Psi_2)_r = (\cos\theta, \sin\theta, 1)$$

$$(\Psi_2)_\theta = (-r\sin\theta, r\cos\theta, 0)$$

$$N_{\Psi_2} = (\Psi_2)_r \times (\Psi_2)_\theta = (-r\cos\theta, -r\sin\theta, r)$$

$$|N_{\Psi_2}| = \sqrt{r^2\cos^2\theta + r^2\sin^2\theta + r^2} = \sqrt{2}\,r \neq 0$$

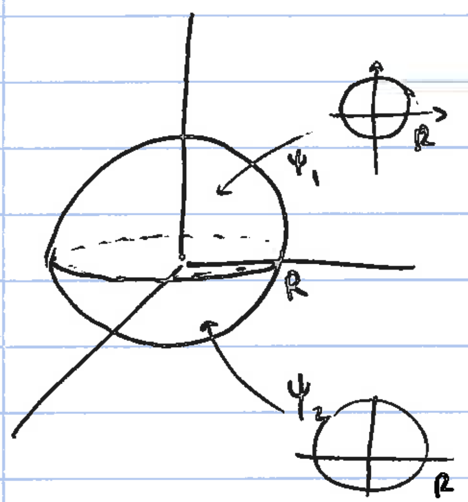
if $r>0$.

$\exists$ No parametrization of the cone about $\vec{0}$, which is regular.

All parametrizations of the cone about $\vec{0}$ will be singular.

That is not the case for the paraboloid, since there are regular parametrizations of the paraboloid about $\vec{0}$, see Ex (2a).

Ex 4 Sphere $x^2+y^2+z^2=R^2$

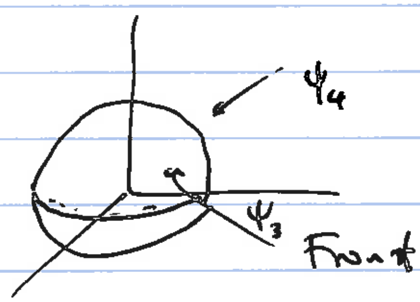


$$\psi_1(u,v) = \left(u, v, \sqrt{R^2-u^2-v^2}\right)$$

$$\psi_2(u,v) = \left(u, v, -\sqrt{R^2-u^2-v^2}\right)$$

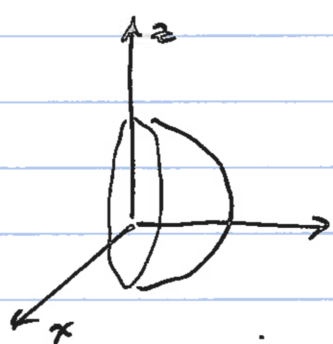
$$D = \{(u,v) \mid u^2+v^2 < R^2\}$$

The domain for all $\psi_1, \psi_2 \dots \psi_6$.



$$\psi_3(u,v) = \left(\sqrt{R^2-u^2-v^2}, u, v\right)$$

$$\psi_4(u,v) = \left(-\sqrt{R^2-u^2-v^2}, u, v\right)$$



$$\psi_5(u,v) = \left(u, \sqrt{R^2-u^2-v^2}, v\right)$$

$$\psi_6(u,v) = \left(u, -\sqrt{R^2-u^2-v^2}, v\right).$$

$$\begin{array}{cccc} & i & j & k \end{array}$$

$$(\psi_1)_u = \left(1, 0, \frac{-2u}{2\sqrt{R^2-u^2-v^2}}\right)$$

$$(\psi_1)_v = \left(0, 1, \frac{-2v}{2\sqrt{R^2-u^2-v^2}}\right)$$

$$N_{\psi_1} = \left(\frac{u}{\sqrt{R^2-u^2-v^2}}, \frac{v}{\sqrt{R^2-u^2-v^2}}, 1\right) \neq \vec{0}$$

Defined on $u^2+v^2<R^2$