Oct 25, 2017

①

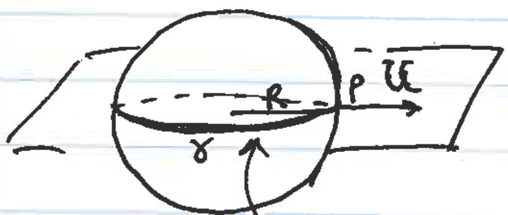
3.1

Ex.

S^2_R. outward normal.

$\parallel$

$\{(x,y,z) \mid x^2+y^2+z^2=R^2\}$



All normal sections are circles of radius R

normal section.

$$k_\gamma(p) = \frac{1}{R}.$$

$$k(u) = -\frac{1}{R}.$$

N_γ pointing inwards] $\cos\theta = -1$

$\vec{u}$ pointing out.] $\theta = \measuredangle(N_\gamma, \vec{u})$

$$[S_p] = \begin{bmatrix} -\frac{1}{R} & 0 \\ 0 & -\frac{1}{R} \end{bmatrix} = -\frac{1}{R}\, Id.$$

Any Basis

$$K_{S^2_R} \equiv \frac{1}{R^2}$$

$$H_{S^2_R} \equiv -\frac{1}{R}.$$ $\rightsquigarrow$ deforming S^2_R in the direction of the given normal $\vec{u}$ will increase area.

If $\tilde{u}$ inward normal $[\tilde{S}_p] = \begin{bmatrix} \frac{1}{R} & 0 \\ 0 & \frac{1}{R} \end{bmatrix}$. $\tilde{K} = \frac{1}{R^2}$

$\tilde{H} = \frac{1}{R}.$

Monkey Saddle

Ex ③ $\quad \Psi(u,v) = (u, v, u^3 - 3uv^2)$

$$\Psi_u = (1, 0, 3u^2 - 3v^2)$$
$$\Psi_v = (0, 1, -6uv)$$

$$\Psi_{uu} = (0, 0, 6u)$$
$$\Psi_{uv} = (0, 0, -6v)$$
$$\Psi_{vv} = (0, 0, -6u)$$

$$\Psi_u \times \Psi_v = (3v^2 - 3u^2, 6uv, 1)$$

$$\Delta^2 = \|\Psi_u \times \Psi_v\|^2 = 9(u^2 + v^2)^2 + 1 = EG - F^2$$

$$U = \frac{\Psi_u \times \Psi_v}{\Delta}.$$

$$E = 1 + (3u^2 - 3v^2)^2 = 1 + 9(u^2 - v^2)$$
$$F = 18uv(v^2 - u^2)$$
$$G = 1 + 36u^2v^2$$

$$l = U \cdot \Psi_{uu} = \frac{1}{\Delta} \begin{vmatrix} \Psi_{uu} \\ \Psi_u \\ \Psi_v \end{vmatrix}_{\substack{3\times 3 \\ \text{det.}}} = \frac{1}{\Delta} \begin{vmatrix} 0 & 0 & 6u \\ 1 & 0 & 3u^2 - 3v^2 \\ 0 & 1 & -6uv \end{vmatrix}$$

$$= \frac{6u}{\Delta}.$$

$$\frac{\Psi_u \times \Psi_v}{\Delta} \cdot \Psi_{uu}$$

$$m = U \cdot \psi_{uv} = \frac{1}{\Delta} \begin{vmatrix} \psi_{uv} \\ \psi_u \\ \psi_v \end{vmatrix} = \frac{1}{\Delta} \begin{vmatrix} 0 & 0 & -6v \\ 1 & 0 & 3u^2-3v^2 \\ 0 & 1 & -6uv \end{vmatrix}$$

$$= -\frac{6v}{\Delta}.$$

$$n = U \cdot \psi_{vv} = \frac{1}{\Delta} \begin{vmatrix} \psi_{vv} \\ \psi_u \\ \psi_v \end{vmatrix} = \frac{1}{\Delta} \begin{vmatrix} 0 & 0 & -6u \\ 1 & 0 & 3u^2-3v^2 \\ 0 & 1 & -6uv \end{vmatrix}$$

$$= -6u/\Delta.$$

$$K = \frac{ln - m^2}{\underbrace{EG - F^2}_{\Delta^2}} = -\frac{36}{\Delta^4}(u^2+v^2) \leq 0$$

$$EG - F^2 = \|\psi_u \times \psi_v\|^2$$

$$K(u,v) = 0 \iff (u,v) = (0,0)$$

$$H = \frac{1}{2} \frac{Gl + En - 2Fm}{EG - F^2}$$

$$= \frac{3u}{\Delta^3}(u^2+v^2)(3v^2-u^2)$$

Prop $\forall\, v, w$ linearly independent in T_pM

1) $S_p(v) \times S_p(w) = K(p)(v \times w)$

2) $S_p(v) \times w + v \times S_p(w) = 2H(p)(v \times w)$

Proof Recall det, trace are independent of choices of basis.

1) $S_p(v) = av + bw$
$S_p(w) = cv + dw$

$$[S_p]_{\{v,w\}} = \begin{bmatrix} a & c \\ b & d \end{bmatrix}$$

$$S_p(v) \times S_p(w) = (a\vec{v} + b\vec{w}) \times (c\vec{v} + d\vec{w})$$

$$= ac\underbrace{(v \times v)}_{0} + ad(v \times w) + bc(w \times v) + bd\underbrace{(w \times w)}_{0}$$

$v \times w = -w \times v$

$$= (ad - bc)(v \times w)$$

$$= \det S_p\,(v \times w)$$

$$= K(p)(v \times w)$$

2) plug into the formula $\longrightarrow$ comes out.