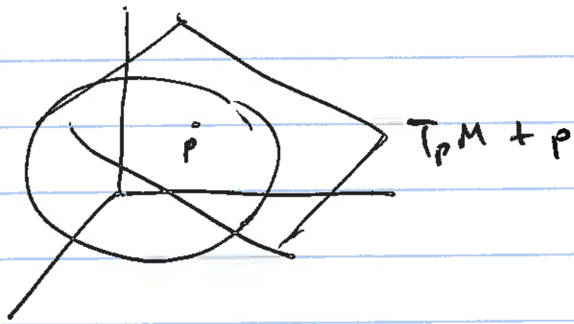
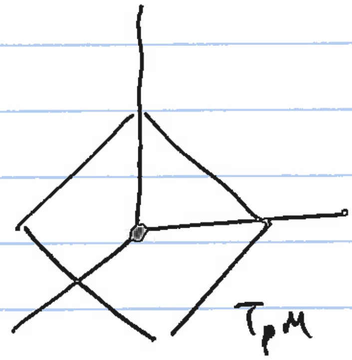
2.2 Continue

T_pM is a linear vector subspace of $\mathbb{R}^3$.



Affine



Linear

Example $\varphi(u,v) = (u^2, v, u^2+v^3)$

Want Tangent plane to φ at $\varphi(2,1) = \underbrace{(4,1,5)}_{p}$

$\varphi_u = (2u, 0, 2u)$ $\quad\quad \varphi_u(2,1) = (4,0,4)$

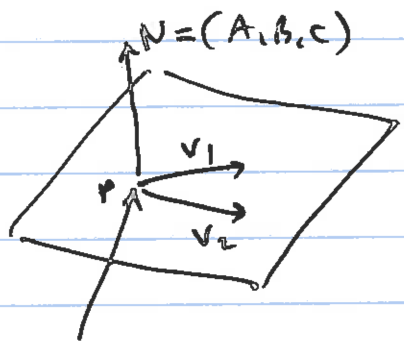
$\varphi_v = (0, 1, 3v^2)$ $\quad\quad \varphi_v(2,1) = (0,1,3)$

$$T_pM = \{ c_1(4,0,4) + c_2(0,1,3) \mid c_1, c_2 \in \mathbb{R} \} \subseteq \mathbb{R}^3$$

$$p + T_pM = \{ \underbrace{(4,1,5) + c_1(4,0,4) + c_2(0,1,3)}_{Ax+By+Cz=D} \mid c_1, c_2 \in \mathbb{R} \}$$

$\boxed{-4x - 12y + 4z = -8}$

$-16 - 12 + 20$



$N = \varphi_u \times \varphi_v (2,1)$

$= (4,0,4) \times (0,1,3)$

$$= \begin{vmatrix} i & j & k \\ 4 & 0 & 4 \\ 0 & 1 & 3 \end{vmatrix} = (-4, -12, 4)$$

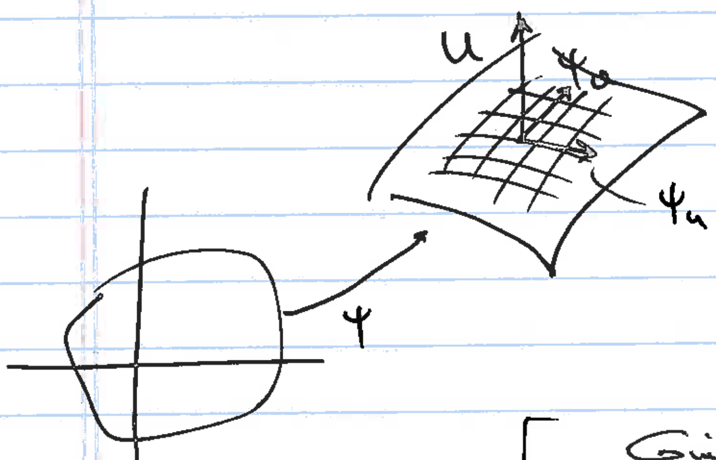
ORIENTATION is a choice of a side of a surface

1. Every regular parametrization $\Psi : D \longrightarrow S$ determines a local orientation

$$N = \Psi_u \times \Psi_v \neq 0.$$

$$U(u,v) = \frac{N}{|N|} = \frac{\Psi_u \times \Psi_v}{|\Psi_u \times \Psi_v|}$$

unit normal associated with the parametrization Ψ.



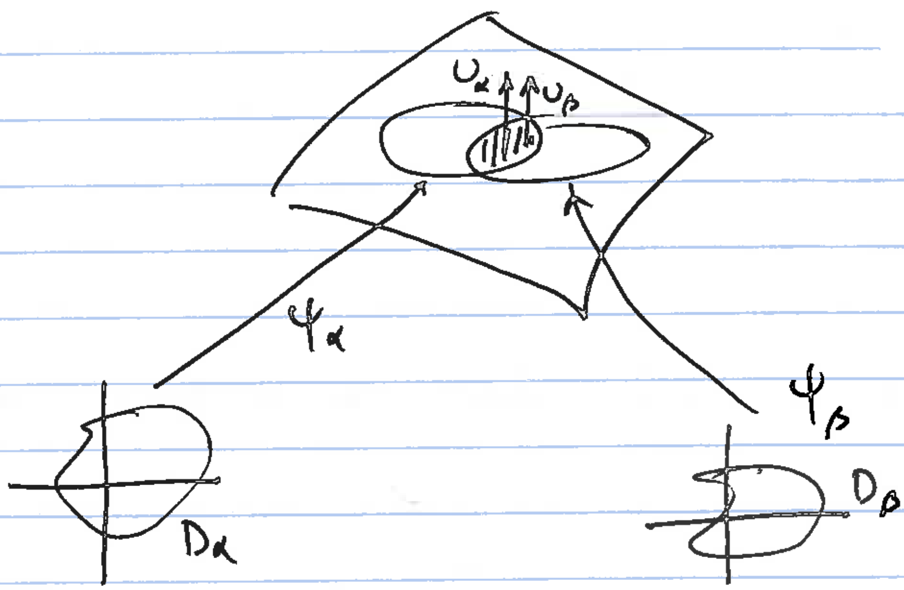
2. Trick : [Given $\Psi(u,v)$ take $\Phi(u,v) = \Psi(v,u)$ for opposite normal

$$U_\Phi(a,b) = \frac{\Phi_u \times \Phi_v}{|\Phi_u \times \Phi_v|}(a,b) = -\frac{\Psi_u \times \Psi_v}{|\Psi_u \times \Psi_v|}(b,a) = -U_\Psi(b,a)$$

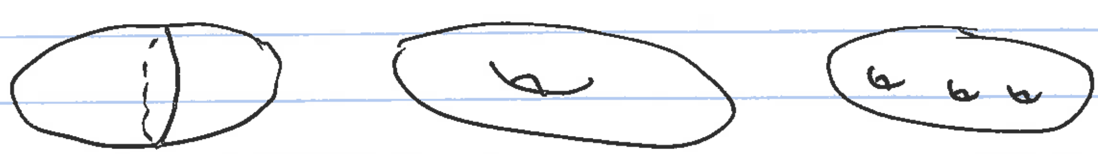
③ **Defn** A regular surface M is called orientable if $\exists$ family of parametrizations

$$\{ \varphi_\alpha : D_\alpha \subseteq \mathbb{R}^2 \to M \mid \alpha \in \Lambda \}$$

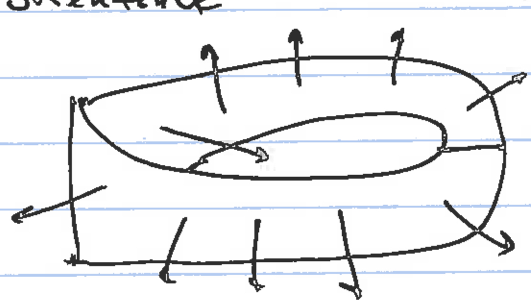
s.t. $\varphi_\alpha(D_\alpha) \cap \varphi_\beta(D_\beta) \neq \phi$.
then $U_{\varphi_\alpha} = U_{\varphi_\beta}$



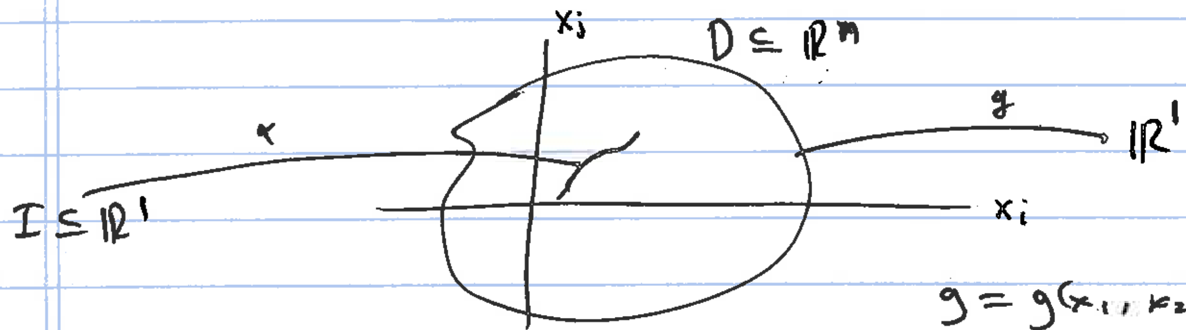
④ Examples of orientable surfaces



Not orientable Mobius Band



Differentiation in $\mathbb{R}^n$
Tangent vectors



$g = g(x_1, x_2, \dots x_n)$

$\alpha(t) = (\alpha_1(t), \dots \alpha_n(t))$

$$\frac{d}{dt} g(\alpha(t))(t_0) =$$

$$= \frac{\partial g}{\partial x_1}(\alpha(t_0)) \cdot \frac{dx_1}{dt}(t_0) + \frac{\partial g}{\partial x_2}(\alpha(t_0)) \cdot \frac{dx_2}{dt}(t_0) + \dots$$

$$\dots + \frac{\partial g}{\partial x_n}(\alpha(t_0)) \cdot \frac{dx_n}{dt}(t_0)$$

$$= \sum_{i=1}^{n} \frac{\partial g}{\partial x_i}(\alpha(t_0)) \cdot \frac{dx_i}{dt}(t_0)$$

$$= (\mathrm{grad}\ g)(\alpha(t_0)) \cdot \alpha'(t_0)$$ where

$$\mathrm{grad}\ g = \left(\frac{\partial g}{\partial x_1}, \frac{\partial g}{\partial x_2}, \dots \frac{\partial g}{\partial x_n}\right)$$

$$\alpha' = (\alpha_1', \alpha_2', \dots \alpha_n')$$