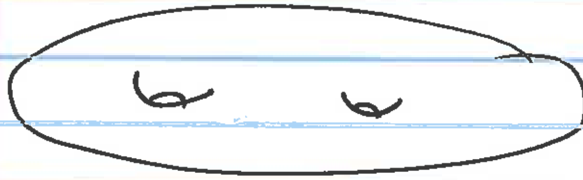
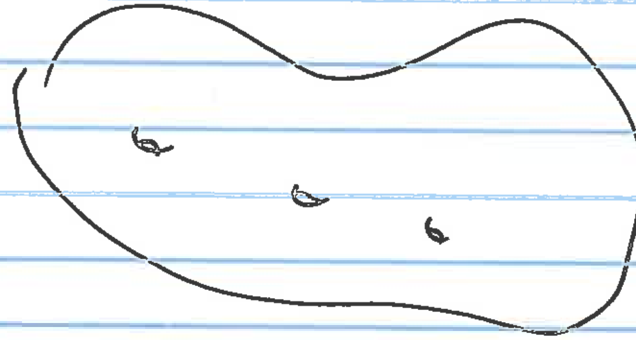
Does there exist $K \equiv$ constant metrics on compact 2-surfaces with no boundary.

M_g $\quad g \geq 2$



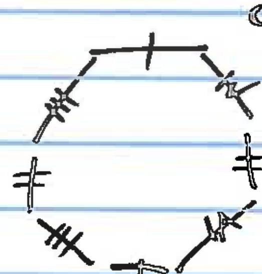


$$\iint_{M_g} K\,dS = \underbrace{(2-2g)}_{\chi(M_g)} \cdot 2\pi$$

$g = 2$ $\quad \iint_{M_g} K\,dS = -4\pi$

$\exists$? a metric on M_g with $K \equiv -1$

<u>Yes</u>.



$\longrightarrow \quad M_2$.

There exist hyperbolic tessellations of the unit disc so that their quotients are M_g, with $K \equiv -1$.

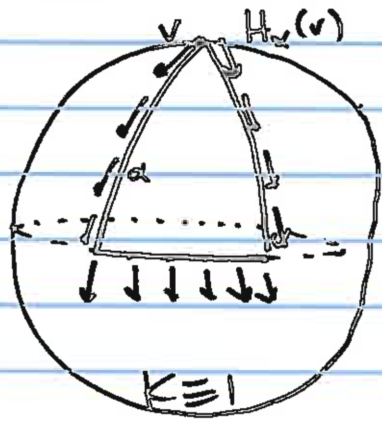
Recall G-B $\quad \int k_g + \sum e_i + \iint_R K\,dS = 2\pi\,\chi(R)$

Other observations about G-B

$$\underbrace{\int k_g + \sum e_i}_{\text{measurements of turning}}$$

continuous vs. discrete.

Holonomy: $\longleftrightarrow \iint_R K\,dS$



Holonomy: the change of the angle when one parallel translates a vector along a closed curve

$$\measuredangle(v, H_\alpha(v)) = \iint_R K\,dS$$

Also see p 283

Theorem Poincaré - Hopf

Let V be a tangent vector field on M^n (compa… $\partial =$…
with isolated zeros, $p_1, \ldots p_\ell$. Then

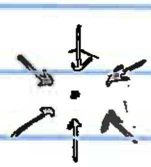
$$\sum_{i=1}^{\ell} I(p_i) = \chi(M)$$

On S^2 $\chi(S^2) = 2$, if there were no zero's of V, then LHS $= 0$
RHS $= 2$. Contradic…

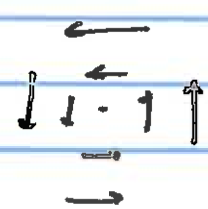
$\Longrightarrow$ On S^2, every vector field must have a 0.
You can't comb the hair on S^2 so that all lay flat.

For $n=2$ Gauss - Bonnet $\Longrightarrow$ Poincaré - Hopf

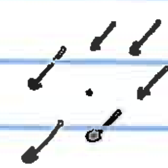
Index $= I(p)$



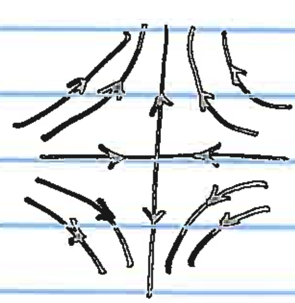
$I = 1$



$I = 1$



$I = 0$



$I = -1.$

$I = 2$