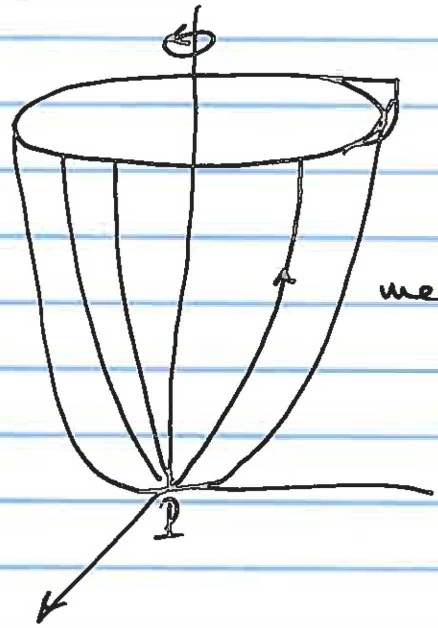
Geodesics of Paraboloid $z = x^2+y^2$



All meridians are geodesic

No parallels are geodesics
(Point curve at P is not considered as a parallel.)

Slanted Geodesics

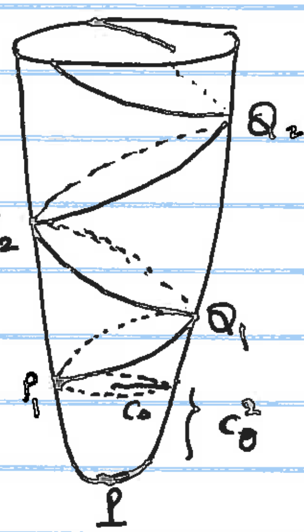
$R(t)\cdot\cos\theta(t) \equiv C_0$.

$C_0 = 0$ (meridians)

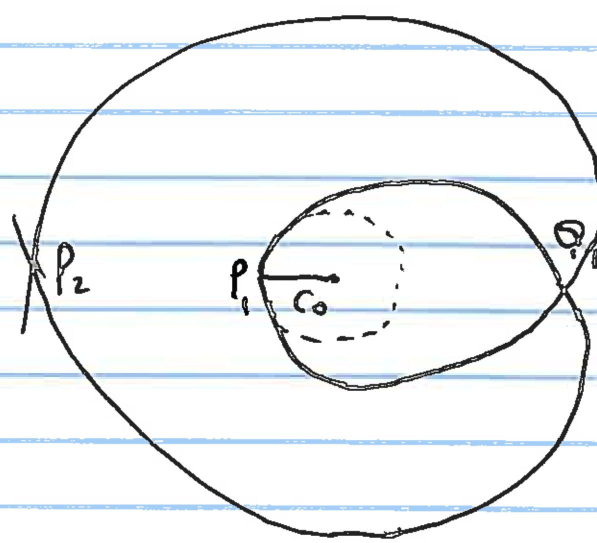
$C_0 > 0$

$R(t) \geqslant C_0$

Side view



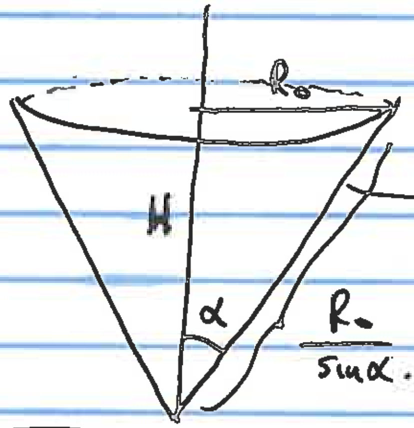
Top view



Prop: ∃ ∞ly many self intersections

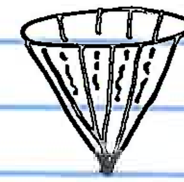
Ex

Geodesics of the Cones



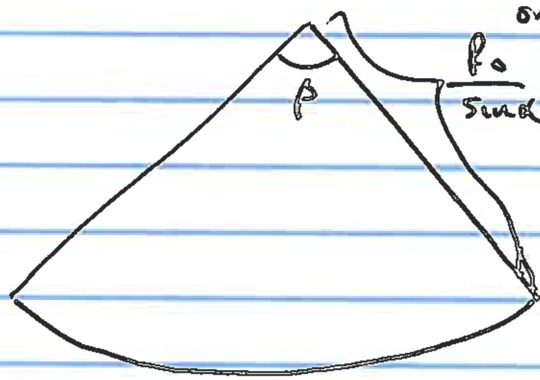
cut cone along a meridian.

(All meridians are geodesics)



open flat on $\mathbb{R}^2$

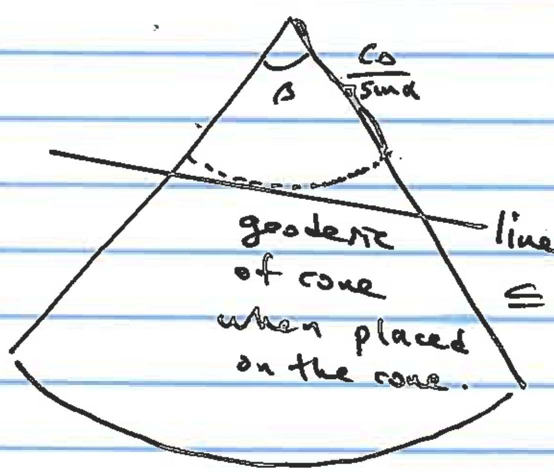
$$z = \cot\alpha \sqrt{x^2+y^2}$$



$\subseteq \mathbb{R}^2$

$$\beta \cdot \frac{R_0}{\sin\alpha} = 2\pi R_0.$$

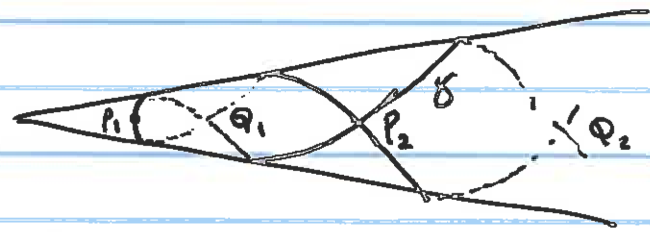
$$\beta = 2\pi \sin\alpha.$$



Slanted geodesic

$$R(t) \cdot \cos\theta(t) = R_0$$

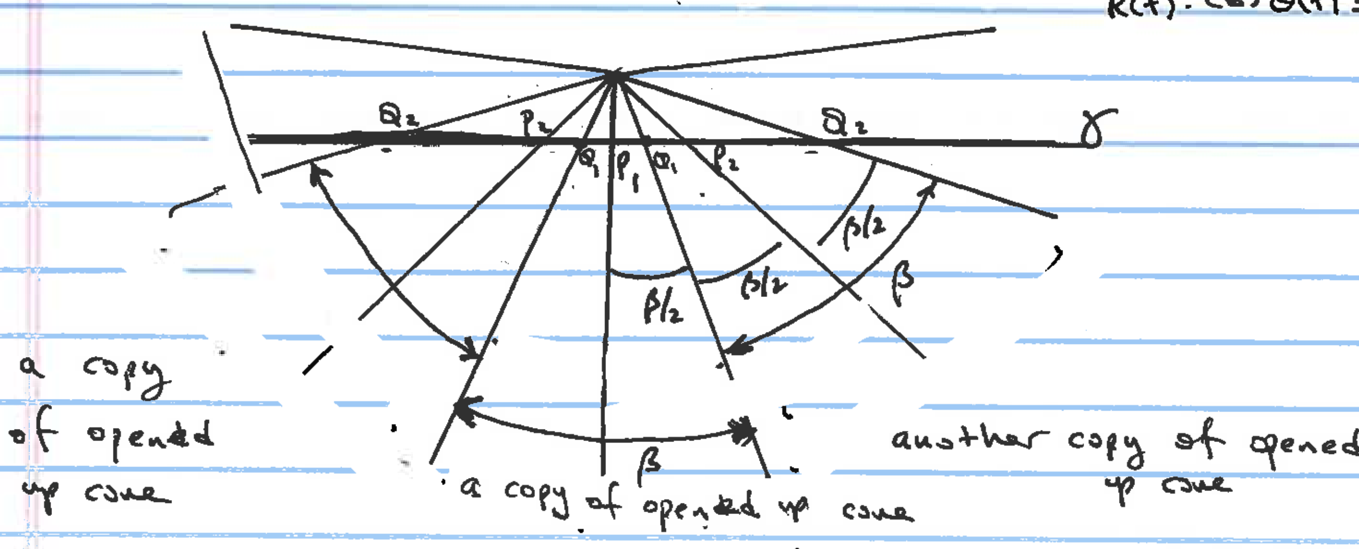
$\subseteq \mathbb{R}^2$



P_i on the front
Q_i on the back.

When does a geodesic of a cone have self intersection? How many?

Cut cone along the meridian
$\psi(u, v_0+\pi)$ if $P_1 = \psi(u_0, v_0)$ ← at lowest $R(t)$ for the geodesic with $R(t)\cdot\cos\theta(t) \equiv c$

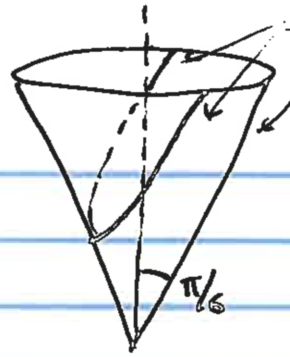


We need # $\beta/2$ angles in $\pi/2$

$$\frac{\pi/2}{\beta/2} = \frac{\pi}{2\pi\sin\alpha} = \frac{1}{2\sin\alpha}.$$

$$n = \left]\!\!\right]\frac{1}{2\sin\alpha}\left[\!\!\right[= \#\text{ self intersections} + \underbrace{1}_{P_1}$$

$\left]\!\!\right]x\left[\!\!\right[$ = smallest integer $n \geq x$



parallel at ∞.

$\alpha = \frac{\pi}{6}$

$\sin\alpha = \frac{1}{2}$

$\frac{1}{2\sin \pi/6} = 1$.

if $\frac{\pi}{6} \le \alpha \le \frac{\pi}{2}$ there are no self intersections.

$\alpha < \frac{\pi}{2}$, there are self intersections but always finitely many

i.e. $]\!]\, \frac{1}{2\sin\alpha} \,[\![- 1$.