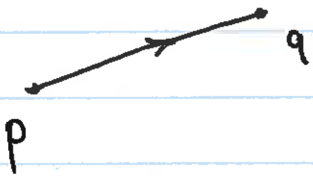
1.1 Examples:

1



$p \neq q$

$$\alpha(t) = p + t(q-p)$$

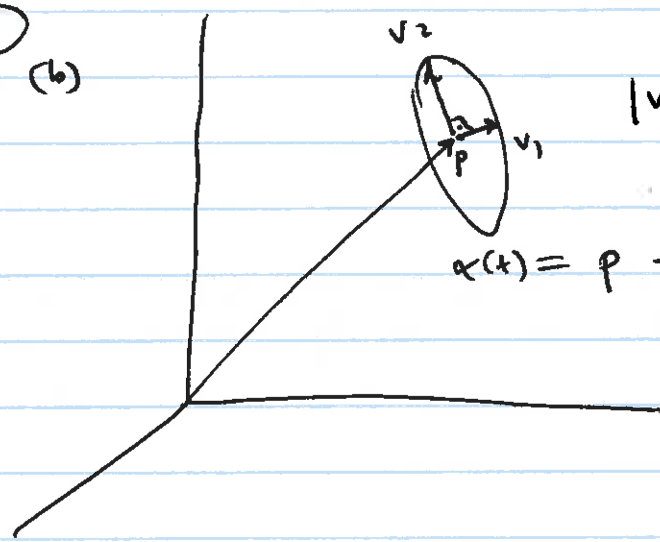
$$= tq + (1-t)p \qquad 0 \le t \le 1$$

Line segment

if $t \in \mathbb{R}$, you'll get a line

Ex 2

(b)



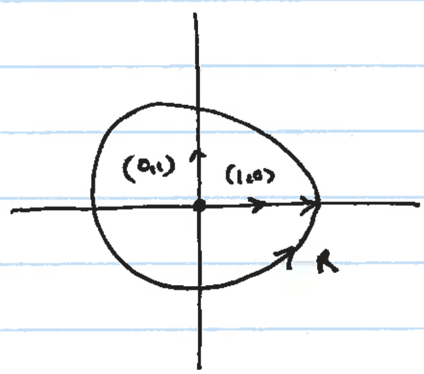
$v_1 \perp v_2$

$|v_1| = |v_2| = R$

circles in $\mathbb{R}^3$

$$\alpha(t) = p + \cos t \cdot v_1 + \sin t \cdot v_2$$

(a)



$(R\cos t, R\sin t)$

$R\cos t\,(1,0) + R\sin t\,(0,1)$

Ex Helix

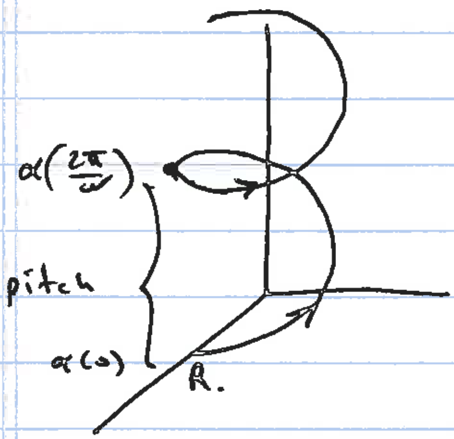
$$\alpha(t) = (R\cos\omega t,\ R\sin\omega t,\ ht)$$

$\omega \neq 0$

Top view:

$\omega > 0$

$\omega < 0$



$$\alpha(0) = (R, 0, 0)$$

$$\alpha\left(\tfrac{2\pi}{\omega}\right) = \Big(R, 0, \underbrace{h \cdot \tfrac{2\pi}{\omega}}_{\text{pitch}}\Big)$$

$$\alpha' = (-\omega R \sin\omega t,\ R\omega\cos t,\ h) \quad \text{velocity}$$

$$\alpha'' = (-\omega^2 R\cos\omega t, -W^2 R \sin t, 0) \quad \text{acceleration}$$

$$|\alpha'| = \sqrt{\omega^2 R^2 + h^2} \quad \text{speed.}$$

Length of one turn i.e. from $\alpha(0)$, $\alpha\left(\frac{2\pi}{\omega}\right)$.

$$= \int_0^{2\pi/\omega} |\alpha'(t)|\,dt = \int_0^{2\pi/\omega} \sqrt{\omega^2 R^2 + h^2}\,dt$$

$$= \frac{2\pi}{\omega}\sqrt{\omega^2 R^2 + h^2}$$

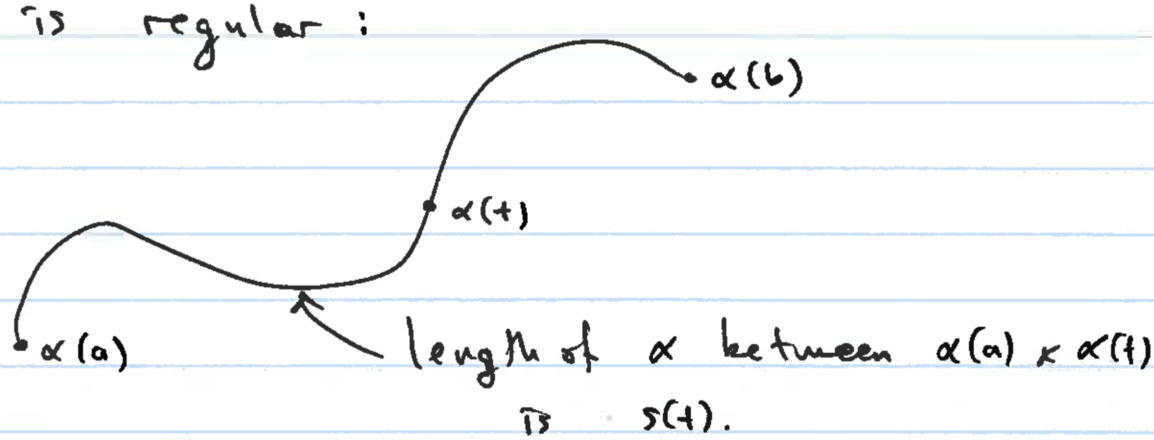
Arclength parameter $s = \int_0^t |\alpha'(u)|\,du = t \cdot \sqrt{\omega^2 R^2 + h^2}$.

Defn Let $\alpha : [a,b] \to \mathbb{R}^n$, be of class C^1,

$$s(t) = \int_a^t |\alpha'(u)|\,du \quad \text{is called the}$$

arclength parameter.

If α is regular:



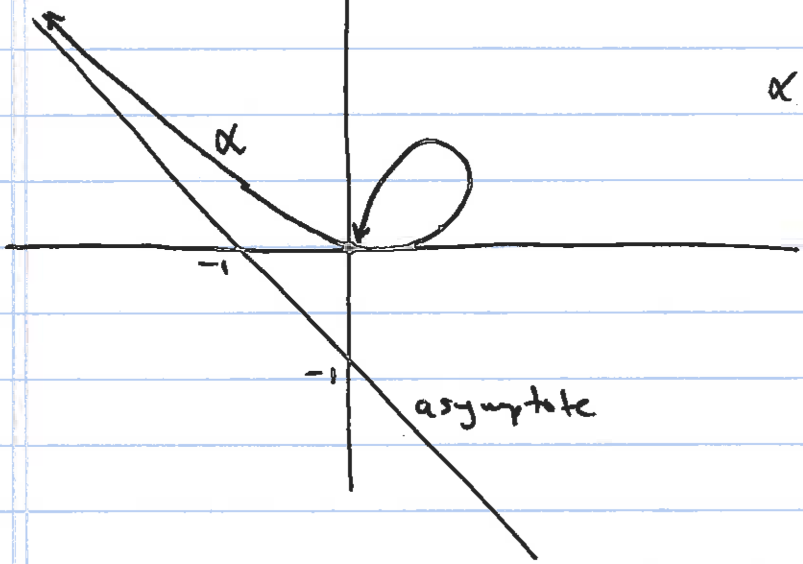
Ex Folium of Descartes

$$\alpha(t) = \left(\underbrace{\frac{3t}{1+t^3}}_{x(t)}, \underbrace{\frac{3t^2}{1+t^3}}_{y(t)} \right) : (-1, \infty)$$

$$\lim_{t\to\infty} \alpha(t) = (0,0) = \alpha(0)$$

$$\lim_{t\to -1^+} \alpha(t) = (-\infty, \infty) ; \quad \lim_{t\to -1^+} \frac{y(t)}{x(t)} = \lim_{t\to -1^+} t = -1 .$$

Folium of Descartes



$$\overset{6}{\underset{\text{Image}}{\shortparallel}}$$

$\alpha(t) : (-1,\infty) \longrightarrow \text{Image}$

1−1, onto,

continuous

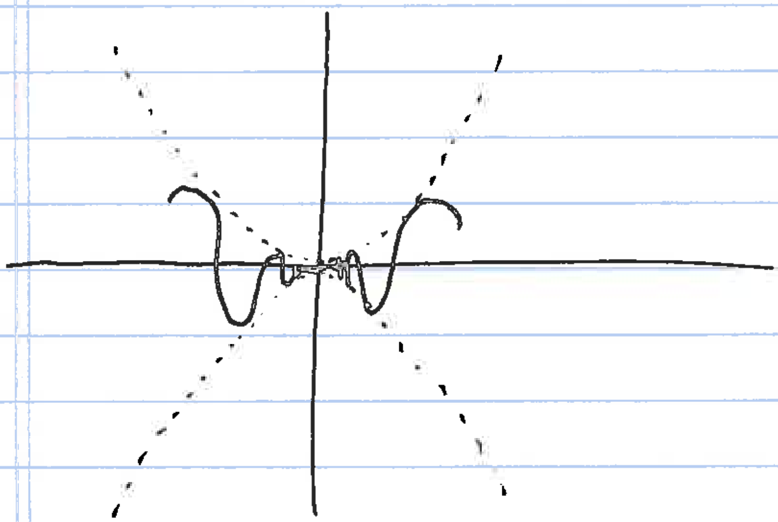
Its inverse (exists)

$6 \longrightarrow (-1,\infty)$

not continuous.

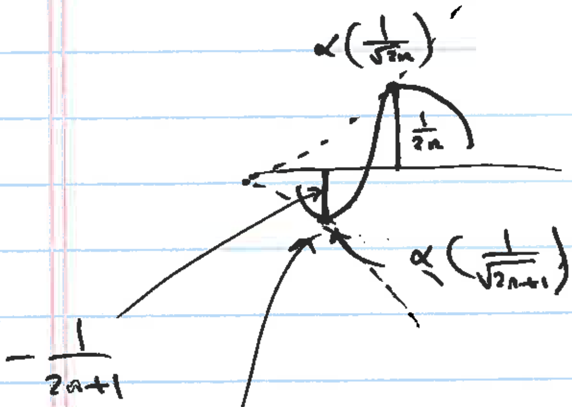
Ex: $\alpha = (t, f(t))$

$$f(t) = \begin{cases} t^2 \cos\frac{\pi}{t^2} & t \neq 0 \\ 0 & t = 0 \end{cases}$$



$\alpha \in \mathcal{D}^1$

$\alpha \notin C^1$



Want $\frac{\pi}{t^2} = 2n\pi$ → its cosine is $+1$

$$t = \frac{1}{\sqrt{2n}}.$$

$$\alpha\left(\frac{1}{\sqrt{2n}}\right) = \left(\frac{1}{\sqrt{2n}}, \frac{1}{2n}\right)$$

Want $\frac{\pi}{t^2} = (2n+1)\pi$. ← its cosine is -1.

$$t = \frac{1}{\sqrt{2n+1}}$$

$$\alpha\left(\frac{1}{\sqrt{2n+1}}\right) = \left(\frac{1}{\sqrt{2n+1}}, \frac{-1}{2n+1}\right)$$

$$L\left(\alpha \Big| \left[\frac{1}{\sqrt{2n+1}}, \frac{1}{\sqrt{2n}}\right]\right) \geq \frac{1}{2n} + \frac{1}{2n+1}.$$

From Calc II: $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n} = \infty$

$$\Rightarrow \text{length}\left(\alpha \big| (0, \varepsilon]\right) = \infty, \ \forall \varepsilon > 0$$

For a C^1 curve $\displaystyle\int_a^b |\alpha'(t)|\, dt < \infty$, for finite a, b.

Example shows that if $\alpha \in \mathcal{D}'$ only, then $\displaystyle\int_a^b |\alpha'(t)|\, dt$ may not be finite.

Exc 13 Hint: