Review + Q/A Session

Oct 1, 2014

1

Exc. 3.1.23 (c) Prove $\underbrace{2^n \le n!}_{P(n)}$ for $n \geq 4$

- $P(4)$: $\underset{16}{\underset{\parallel}{2^4}} \le \underset{24}{\underset{\parallel}{4!}}$ is true. (Base)

To prove $\forall k \geq 4 \; (P(k) \Longrightarrow P(k+1))$ (induction)

- induction hypothesis $0 < 2^k \le k!$ $\quad P(k)$

$0 < 2 \le k+1$ since $k \geq 4$

$2^{k+1} \le k!(k+1) = (k+1)!$

$2^{k+1} \le (k+1)!$ $\quad P(k+1)$

By Thm of Math induction $\forall n \geq 4 \quad 2^n \le n!$

Exc 1.4 # 14 p 36

Prove $\frac{x}{x-2} \leq 3 \implies x < 2$ or $x \geq 3$

$$p \implies (q \text{ or } r)$$

suffices to prove $(p \text{ and } \sim q) \implies r$

Want to prove: $\left(\frac{x}{x-2} \leq 3 \text{ and } x \geq 2 \right) \implies x \geq 3$

Proof

$$\frac{x}{x-2} \leq 3$$

$$x \geq 2$$

$x \neq 2$ since $\frac{x}{x-2}$ is defined in order to be ≤ 3.

$$x - 2 > 0$$

$$\left(\frac{x}{x-2}\right)(x-2) \leq 3 \cdot (x-2)$$

$$x \leq 3x - 6$$

$$6 \leq 2x$$

$$3 \leq x \quad . \quad \#$$

Exc 5 (2018 MT) T/F

$\sim\ \exists x \in \mathbb{R} \ni \forall y \in \mathbb{R}\ (x+y<5 \Rightarrow \exists z \in \mathbb{R}\ (xy^2<1\ \&\ x>z))$

$\forall x \in \mathbb{R}\ \exists y \in \mathbb{R}\ (x+y<5 \text{ and } \forall z \in \mathbb{R} (xy^2 \geq 1 \text{ OR } x \leq z))$

Compare (a) False

1b

True

$$\begin{array}{ccc} \mathbb{N} & \xrightarrow[f]{bij} & \mathbb{Z} \\ \text{Id} \Big\| & & \Big\downarrow g \circ f^{-1} \text{ bijection.} \\ \mathbb{N} & \xrightarrow[g]{bij} & \mathbb{Q} \end{array}$$

1c False since both (i) & (ii) are true.

1d False checking $n \notin A$ will tell nothing about $n \in A$.

1e True $|x|-|y| \leq \big|\,|x|-|y|\,\big| \leq |x-y|$

Exc #6a p121

Ex 6a 3.2 p 121

Know $|x+y| \leq |x| + |y| \quad \forall x,y \in \mathbb{R}$ Thm 3.2.10

Want To show $|x| - |y| \leq |x-y| \quad \forall x,y \in \mathbb{R}$.

Trick

$$|x| = |\underbrace{x-y}_{a} + \underbrace{y}_{b}| = |a+b| \leq |a| + |b|$$

$$\leq |x-y| + |y|$$

$$|x| \leq |x-y| + |y|$$

$$|x| - |y| \leq |x-y|.$$

similarly:

$$-(|x|-|y|) = |y| - |x| \leq |y-x| = |x-y|$$

both $|x| - |y| \leq |x-y|$
and $-(|x|-|y|) \leq |x-y|$

So. $||x|-|y|| \leq |x-y|$.

Midterm 2006 # 3

HW

Also Exc 2.3.15g

$f : A \to B$

$C \subseteq A, \quad D \subseteq B.$

Prove that $\quad f^{-1}(B - D) = A - f^{-1}(D)$

$\forall x \in A$

$$x \in f^{-1}(B-D) \iff f(x) \in B \setminus D$$

$$\iff \underbrace{f(x) \in B}_{\text{always true (since } f: A \to B)} \text{ and } f(x) \notin D.$$

$$\iff f(x) \notin D \text{ and } x \in A$$

$$\iff x \notin f^{-1}(D) \text{ and } x \in A$$

$$\iff x \in A - f^{-1}(D)$$

Midterm 2018 (3b)

f surjective $\Longrightarrow$ $f(f^{-1}(D)) = D.$

* Can Assume $f(f^{-1}(D)) \subseteq D.$ true for all functions
Thm 2.3.16b

Want to show $D \subseteq f(f^{-1}(D))$

Let $y \in D$ be an arbitrary element of D

f surjective $\Rightarrow \exists\, x \in A$ s.t. $f(x) = y$

** $f(x) = y \in D.$

$x \in f^{-1}(D)$

$y = f(x) \in f(f^{-1}(D)).$

$\forall y \in D,\ y \in f(f^{-1}(D))$

$D \subseteq f(f^{-1}(D)).$

$(*)$ & $(**) \Longrightarrow D = f(f^{-1}(D))$