

Review + Q/A Session

Oct 1, 2014

①

Exc. 3.1.23 (c) Prove $\underbrace{2^n \leq n!}_{P(n)}$ for $n \geq 4$

- $P(4) : \underset{16}{2^4} \leq \underset{24}{4!}$ is true. (Base)

To prove $\forall k \geq 4$ ($P(k) \implies P(k+1)$) (induction)

- induction hypothesis $0 < 2^k \leq k!$ $P(k)$
 $0 < 2 \leq k+1$ since $k \geq 4$
 $2^{k+1} \leq k!(k+1) = (k+1)!$

$$2^{k+1} \leq (k+1)! \quad P(k+1)$$

By Thm of Math induction $\forall n \geq 4$ $2^n \leq n!$

(2)

Exc 1.4 # 14 p 36

Prove $\frac{x}{x-2} \leq 3 \implies x < 2$ or $x \geq 3$

$p \implies (q \text{ or } r)$

Suffices to prove $(p \text{ and } \neg q) \implies r$

Want to prove $\left(\frac{x}{x-2} \leq 3 \text{ and } x \geq 2 \right) \implies x \geq 3$

Proof

$$\frac{x}{x-2} \leq 3$$

$$x \geq 2$$

$x \neq 2$ since $\frac{x}{x-2}$ is defined in order to be ≤ 3 .

$$x-2 > 0$$

$$\left(\frac{x}{x-2} \right) (x-2) \leq 3 \cdot (x-2)$$

$$x \leq 3x - 6$$

$$6 \leq 2x$$

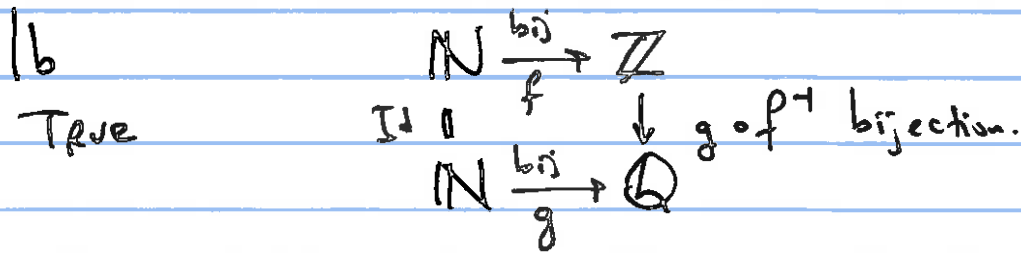
$$3 \leq x \quad \#$$

Exc 5 (2018) T/F
MT

2 $\exists x \in \mathbb{R} \Rightarrow \forall y \in \mathbb{R} (x+y < 5 \Rightarrow \exists z \in \mathbb{R} (x^2 < 1 \wedge x > z))$

$\forall x \in \mathbb{R} \exists y \in \mathbb{R} (x+y < 5 \text{ and } \forall z \in \mathbb{R} (x^2 \geq 1 \text{ or } x \leq z))$

Compare (a) False



1c False since both (i) & (ii) are true.

1d False checking $n \notin A$ will tell nothing about $n \in A$.

1e True $|x| - |y| \leq |(x) - |y|| \leq |x - y|$

Ex 6a 3.2 p 121

Know $|x+y| \leq |x|+|y| \quad \forall x,y \in \mathbb{R}$ Thm 3.2.10

Want To show $||x|-|y|| \leq |x-y| \quad \forall x,y \in \mathbb{R}$.

Trick

$$|x| = \underbrace{|x-y|}_{a} + \underbrace{|y|}_{b} = |a+b| \leq |a|+|b| \\ \leq |x-y| + |y|$$

$$|x| \leq |x-y| + |y|$$

$$|x| - |y| \leq |x-y|.$$

Similarly:

$$-(|x|-|y|) = |y|-|x| \leq |y-x| = |x-y|$$

$$\text{both } |x|-|y| \leq |x-y|$$

$$\text{and } -(|x|-|y|) \leq |x-y|$$

$$\text{So, } ||x|-|y|| \leq |x-y|.$$

Midterm 2006 #3

HW
Also Ex 2.3.15g

$$f: A \rightarrow B$$

$$C \subseteq A, D \subseteq B.$$

Prove that $f^{-1}(B - D) = A - f^{-1}(D)$

$\forall x \in A$

$$x \in f^{-1}(B - D) \iff f(x) \in B - D$$

$$\iff \underbrace{f(x) \in B}_{\text{always true (since } f: A \rightarrow B)} \text{ and } f(x) \notin D.$$

$$\iff f(x) \notin D \text{ and } x \in A$$

$$\iff x \notin f^{-1}(D) \text{ and } x \in A$$

$$\iff x \in A - f^{-1}(D)$$

$$f \text{ surjective} \implies f(f^{-1}(D)) = D.$$

* Can Assume $f(f^{-1}(D)) \subseteq D$. true for all functions
Thm 2.3.16b

Want to show $D \subseteq f(f^{-1}(D))$

Let $y \in D$ be an arbitrary element of D

f surjective $\implies \exists x \in A$ s.t.

$$f(x) = y$$

$$f(x) = y \in D.$$

$$x \in f^{-1}(D)$$

$$y = f(x) \in f(f^{-1}(D)).$$

$$\forall y \in D, y \in f(f^{-1}(D))$$

$$D \subseteq f(f^{-1}(D)).$$

$$(*) \wedge (***) \implies D = f(f^{-1}(D))$$