Nov 5, 2018

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3770 MT2 Fall 2018 Grade Cuts

	102		
A	93	7/30	23%
A-	87		
B+	80	12/30	40%
B	74		
B-	68		
C+	62	10/30	33%
C	56		
C-	50		
D+	45		
D	40		
D-	35		
F			

<u>FINAL EXAM</u>: Thursday
Dec 13
8-10 pm
106 GILH (Gilmore Hall)

Nov 5, 2018

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Chap V

5.1

Defn Let $f: D \to \mathbb{R}$, c be an accumulation pt of D. We write

$$L = \lim_{x \to c} f(x) \quad \text{if}$$

$$\forall \varepsilon > 0 \ \exists \delta > 0 \ \forall x \in D \quad (0 < |x-c| < \delta \Rightarrow |f(x) - L| < \varepsilon .)$$

Ex 1 Prove $\lim_{x \to 3} 2x + 4 = 10$

We want to show

$$\forall \varepsilon > 0 \ \exists \delta > 0 \text{ s.t. } 0 < |x-3| < \delta \Rightarrow |(2x+4) - 10| < \varepsilon .$$

Prep/scrap. $0 < |x-3| < \delta \Rightarrow |2x+4-10|$

$$= |2x-6|$$

$$= 2\underbrace{|x-3|}_{< 2\delta} < \varepsilon .$$

Proof Let $\varepsilon > 0$ be given. Choose $\delta = \frac{\varepsilon}{2} > 0$ then

$\forall x \in \mathbb{R} \quad 0 < |x-3| < \delta \Rightarrow$

$f(x) = 2x+4 : \mathbb{R} \to \mathbb{R}$.

$$|f(x) - L| = |2x + 4 - 10| = |2x-6| = 2|x-3| < 2\delta = \varepsilon .$$

Prove

Ex 2 $\lim_{x\to 1} x^2+x+7=9$

WTS $\forall \varepsilon>0\ \exists \delta>0\ \forall x\in\mathbb{R}\ \left(0<|x-1|<\delta \implies |x^2+x+7-9|<\varepsilon\right)$

Scrap $|x^2+x+7-9| = |x^2+x-2| = |(x-1)(x+2)|$

$= |\underbrace{x-1}_{<\delta}||x+2| \le 4|x-1| < \varepsilon$

take $\delta=\frac{\varepsilon}{4}$.

x near 1.

if $\frac{1}{2}<x<\frac{3}{2}$

$2\frac{1}{2}<x+2<3\frac{1}{2}$

$|x+2|\le 4$

Proof Let $\varepsilon>0$ be given

choose $\delta=\min\left(\frac{\varepsilon}{4},\frac{1}{2}\right)>0$

$\forall x\in\mathbb{R}=$ domain of $(f(x)=x^2+x+7)$

if $|x-1|<\delta\le\frac{1}{2}$

$-\frac{1}{2}<x-1<\frac{1}{2}$

$2\frac{1}{2}<x+2<3\frac{1}{2}$

$|x+2|\le 4$

$\forall x\in\mathbb{R}\ 0<|x-1|<\delta \implies$

$|f(x)-L| = |x^2+x+7-9| = |x^2+x-2|$

$= |(x-1)||x+2| \le |x-1|\cdot 4 < \delta\cdot 4 \le \varepsilon$

Ex 3 Prove $\lim_{x\to 2} \frac{x^2+5x-14}{x-2} = 9$

Proof If $x \neq 2$, $\frac{x^2+5x-14}{x-2} = \frac{(x-2)(x+7)}{x-2} = x+7$.

$x-2 \neq 0$

$\forall \varepsilon > 0 \ \exists \delta = \varepsilon$

If $x \in \mathbb{R}$, $0 < |x-2| < \delta$ then

$$\left| \frac{x^2+5x-14}{x-2} - 9 \right| = \left| \frac{(x-2)\cdot(x+7)}{x-2} - 9 \right|$$

$$= |x+7-9| = |x-2| < \delta$$

Ex 4 Show that $\lim_{x\to 2} x^3 = 8$

If $1 < x < 3$,

then $0 < x^2+2x+4 < 9+6+4$

$|x^2+2x+4| < 19$

Let $\varepsilon > 0$ be given, choose $\delta = \min\left(1, \frac{\varepsilon}{19}\right)$

Let $x \in \mathbb{R}$ domain of x^3 s.t. $0 < |x-2| < \delta$

$$|x^3-8| = |(x-2)(x^2+2x+4)| < \delta \cdot 19 \leq \varepsilon.$$

CHARACTERIZATIONS of Limit

Open set (1) charact.

Thm Let $f: D \to \mathbb{R}$, $c \in D'$. Then

$$\lim_{x \to c} f(x) = L$$

$\iff$ $\forall$ open neighborhood V of L $\exists$ a deleted open neighborhood of c s.t. $f(U \cap D) \subseteq V$

Idea:

$$\lim_{x\to c} f(x) = L \iff \forall \varepsilon > 0\ \exists \delta > 0\ \forall x \in D$$

$$\underbrace{0 < |x-c| < \delta} \implies |f(x) - L| < \varepsilon$$

$-\delta < x - c < \delta$, $x \neq c$

$c - \delta < x < c + \delta$

$-\varepsilon < f(x) - L < \varepsilon$

$L - \varepsilon < f(x) < L + \varepsilon$

$$\forall x \in D\ \big(\ \underbrace{x \in N^*(c,\delta) \implies f(x) \in N(L,\varepsilon)}$$

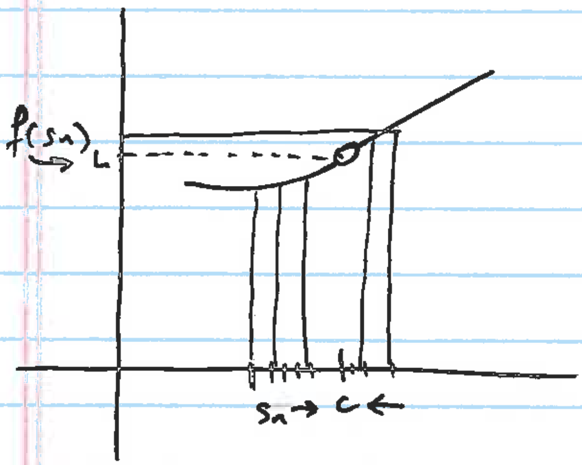
$$f(\underbrace{D \cap \underbrace{N^*(c,\delta)}_{U}}_{D \cap U}) \subseteq \underbrace{N(L,\varepsilon)}_{V}$$

Sequential characterization

Thm Let $f : D \to \mathbb{R}$, $c \in D'$

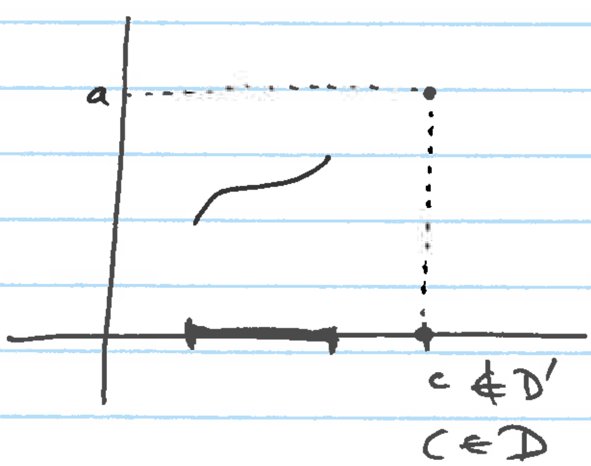
$\lim_{x \to c} f(x) = L \iff$

$\forall$ sequence $(s_n) \subseteq D$, with $s_n \to c$, $s_n \neq c \ \forall n$, one has $\lim_{n\to\infty} f(s_n) = L$.



Proof on Wednesday.

Remark: $c \in D'$ is necessary for limit calculations



$\lim_{x\to c} f(x)$ doesn't make sense since there is no $x \in D$ near/about c, s.t. $x \neq c$ in $\lim_{\underset{\sim}{x \to c}}$

$x \neq c$ required