Oct 29, 2018

Reminders

Midterm 2 Friday in class

Review Session Wednesday 7:30-9:00

110 MLH

To Conclude 3.4

p141 Exc. 7

a) $P = \{\frac{1}{n} : n \in \mathbb{N}\} \subseteq S = \{0\} \cup \{\frac{1}{n} : n \in \mathbb{N}\}$

P is not closed $\quad$ S closed

P is the isolated pts of S.

d) $S = (0,1) \cup (1,2)$ open

$\bar{S} = \operatorname{cl} S = [0,2]$, $\quad \operatorname{int} \bar{S} = (0,2) \neq S$

f) $S = \mathbb{Q}$ $\quad$ $\operatorname{bd} \mathbb{Q} = \mathbb{R}$

$\operatorname{bd}(\operatorname{bd} \mathbb{Q}) = \operatorname{bd} \mathbb{R} = \emptyset \neq \operatorname{bd} \mathbb{Q} = \mathbb{R}$

h)

$S = (0,1)$ $\quad$ $S \cap T = \emptyset$ $\quad$ $\operatorname{bd} S \cap T = \emptyset$.

$T = (1,2)$

$\operatorname{bd} S = \{0,1\}$

$\operatorname{bd} T = \{1,2\}$

$\operatorname{bd} S \cap \operatorname{bd} T = \{1\} \neq \emptyset$.

Another Ex.

$$\underbrace{(0,2)}_{S} \cap \underbrace{(1,3)}_{T} = (1,2) = S \cap T$$

$\operatorname{bd} S \cap T = \{1,2\}$.

$\operatorname{bd} S = \{0,2\}$ $\quad$ $\operatorname{bd} S \cap \operatorname{bd} T = \emptyset$.

$\operatorname{bd} T = \{1,3\}$

4.4 * Compactness* (only my notes)

4.4 Subsequences.

Ex $\left(\frac{1}{n}\right)$ $\quad 1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \cdots$

A subsequence $\quad 1 \quad , \quad \frac{1}{4} \quad , \quad \frac{1}{9} \quad , \quad \frac{1}{16}$

$n_k = k^2$ procedure/formula describing the subsequence

$$\frac{1}{n} \longrightarrow \frac{1}{n_k} = \frac{1}{k^2}$$

$1/k^2$ is a subsequence of $\frac{1}{n}$.

Def Given a sequence (s_n) and let $n_1 < n_2 < n_3 < n_4 < \cdots < n_k < n_{k+1} < \cdots$ be a sequence in $\mathbb{N}$, $n_k \in \mathbb{N}$.

Then $(s_{n_k})_{k=1}^{\infty}$ is called a subsequence of (s_n).

Ex

$s_n = (-1)^n$

$n_k = 2k$

$s_{n_k} = (-1)^{2k} = 1$

$s_n \quad -1, 1, -1, +1, -1, 1, -1, 1, -1, 1$

$s_{n_k} \quad 1 \quad 1 \quad 1 \quad 1 \quad 1$

(3)

(b)

$$S_n = \frac{n^2}{n+1} + (-1)^n.$$

$$n_k = k^3$$

$$S_{n_k} = \frac{k^6}{k^3+1} + (-1)^{k^3}$$

Lemma: $n_k \geq k$.

Proof $1 \leq n_1 < n_2 < n_3 < n_4 < \cdots$ given.

$n_k \in \mathbb{N}$

Induction: $n_1 \geq 1$ since $n_1 \in \mathbb{N}$.

Assume $n_k \geq k$

To show $n_{k+1} \geq k+1$.

$n_{k+1} > n_k \geq k$

$k, n_{k+1} \in \mathbb{N}$, $n_{k+1} > k$.

$n_{k+1} \geq k+1$

since there are no integers between k & $k+1$.

* **Prop** Let (s_n) be a sequence.

$s_n \to L$ as $n \to \infty$ $\iff$ Every subsequence (s_{n_k}) of (s_n) $s_{n_k} \to L$ as $k \to \infty$

Proof $\Leftarrow$: obvious since every sequence is a subsequence of itself.

$\Rightarrow$: $\lim_{n\to\infty} s_n = L$ given.

$\forall \varepsilon > 0 \ \exists N \ \forall n \geq N \ |s_n - L| < \varepsilon$

since $n_k \geq k \ \forall k$

if $k \geq N$ then $n_k \geq N$

So $\forall \varepsilon > 0 \ \exists N$ as above $\forall k \geq N$ $n_k \geq N$

$|s_{n_k} - L| < \varepsilon$.

$\lim_{k\to\infty} s_{n_k} = L$.

Ex $(-1)^n$ is divergent since

$n_k = 2k$ $\quad s_{n_k} = (-1)^{2k} = +1 \longrightarrow 1$.

$n_\ell = 2\ell + 1$ $\quad s_{n_\ell} = (-1)^{2\ell+1} = -1 \longrightarrow -1$.

Hence $(-1)^n$ has two subsequences with different limits.

Prop (*) above $\Longrightarrow$ $(-1)^n$ is divergent.

COMPACTNESS (Skip 3.5) use class notes only

Defn
A set $A \subseteq \mathbb{R}$ is called compact
(3.5) if Every open cover of A has a finite subcover.

Heine-Borel Thm For every subset $A \subseteq \mathbb{R}$:

A is compact $\Longleftrightarrow$ $\underbrace{A \text{ is closed \& bounded.}}$
(defined as above)

via Heine Borel Thm
This is our defn of
compactness in $\mathbb{R}$.

Defn A set $A \subseteq \mathbb{R}$ is called sequentially compact
if every sequence in A has a convergent
subsequence whose limit is in A.

Essential results for proving important Theorems of Calculus.

Thm 1 Bolzano-Weierstrass Every infinite and bounded
subset A of $\mathbb{R}$ has an accumulation pt. in $\mathbb{R}$,
i.e. $A' \neq \phi$.

Thm 2 Every bounded sequence in $\mathbb{R}$ has a convergent
subsequence.

Thm 3 Let $A \subseteq \mathbb{R}$.

A is sequentially compact $\Longleftrightarrow$ A is closed & bounded
(compact)