Sept 24, 2018

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MIDTERM 1

Friday Sept 28

Lecture notes, HW, Textbook.

1.1, 1.2, 1.3, 1.4

2.1, 2.3, 2.4*

3.1, 3.2*

2.4 Only lecture/HW

3.2 Only lecture/HW

Review Session 6:00 – 7:20 214 MLH

Wed 9/26

Geometric meaning of $\lim_{n\to\infty} s_n = L$

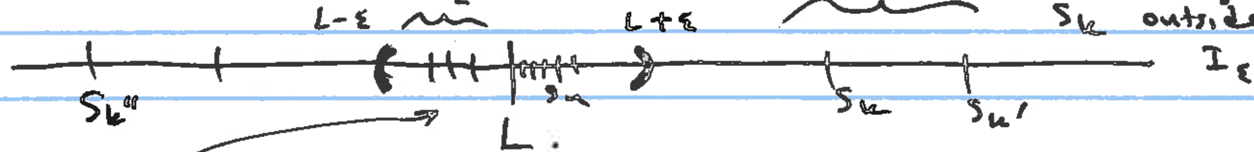
Defn $\forall \varepsilon > 0\ \exists N \in \mathbb{N}\ \forall n \geq N \quad |s_n - L| < \varepsilon.$

$\forall \varepsilon > 0\ \exists N \in \mathbb{N}\ \forall n \in \mathbb{N}\ (n \geq N \implies |s_n - L| < \varepsilon)$

Defn $\implies$ There must be finitely many S_n outside I_ε

interval $I_\varepsilon = (L-\varepsilon, L+\varepsilon)$

only finitely many S_k outside I_ε

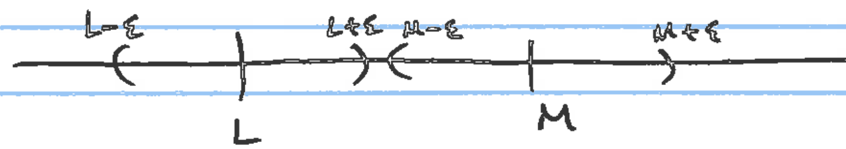


$$|s_n - L| < \varepsilon \iff L - \varepsilon < s_n < L + \varepsilon$$

necessary but not sufficient $\rightarrow$ Infinitely many s_n needs to be inside I_ε

Prop If $\lim_{n\to\infty} s_n = L$ and $\lim_{n\to\infty} = M$, then $L = M$.

Proof Suppose $L \neq M$.



Choose $\varepsilon = \dfrac{|L-M|}{2}$

$\forall \varepsilon > 0\ \exists N_1\ \forall n \in \mathbb{N}\ (n \geq N_1 \implies |s_n - L| < \varepsilon)$

$\forall \varepsilon > 0\ \exists N_2\ \forall n \in \mathbb{N}\ (n \geq N_2 \implies |s_n - M| < \varepsilon)$

If $N = \max(N_1, N_2)$ then

$\forall n \geq N \quad |s_n - L| < \varepsilon$, and

$|s_n - M| < \varepsilon$

Choose any $m \geq N$

$$2\varepsilon = |L - M| = |L - s_m + s_m - M|$$

$$\leq |L - s_m| + |s_m - M| < \varepsilon + \varepsilon = 2\varepsilon$$

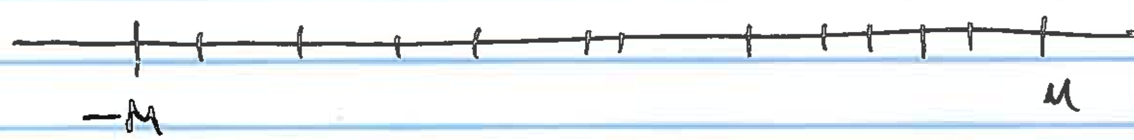
$0 < 2\varepsilon < 2\varepsilon$ Contradiction.

Hence $L = M$.

Defn A sequence (s_n) is called bounded if $\exists\, M \in \mathbb{R}$ s.t. $\forall n$ $|s_n| \leq M$.

i.e.

$\exists\, M > 0$ s.t. $\{s_n\} \subseteq [-M, M]$.



Prop Any convergent sequence is bounded.

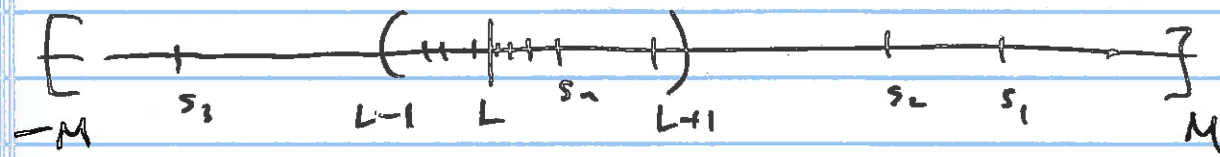
Caution Converse is false.

$s_n = (-1)^n$ bounded but not convergent

Prop. Convergent $\Longrightarrow$ bounded

Proof $\lim s_n = L$

Idea



$\varepsilon = 1$

Proof Let $\varepsilon = 1$ be given $\exists N$ $\forall n \in \mathbb{N}_{\wedge n \geq N}$ $|s_n - L| < 1$.

$$L - 1 < s_n < L + 1$$

$$\forall n \geq N \Longrightarrow |s_n| < |L| + 1$$

Let $M = \max\left(|s_1|, |s_2|, |s_3|, \ldots, |s_k|, \ldots, |s_N|, |L| + 1\right)$

$\forall n \in \mathbb{N}$ **Case 1** $n < N$, $|s_n| \leq M$

Case 2 $n \geq N$ $|s_n - L| < 1$

$\Longrightarrow |s_n| \leq |L| + 1 \leq M$

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(*) Important (We will use 9a, 9c in the future.)

p 170 Exc 9 (b) If $|s_n|$ converges, then (s_n) converges

FALSE $s_n = (-1)^n \not\to$ any number.

$|s_n| = 1 \to 1$.

Exc 4.1.9 p170

9a $s_n \to s \implies |s_n| \to |s|$.

Proof Reverse Δ-inequality: $\big||a|-|b|\big| \le |a-b|$.

$$|s_n - s| \geqslant \big||s_n| - |s|\big| \quad \forall n.$$

Hypothesis $s_n \to s$:

Know $\forall \varepsilon > 0 \ \exists N_1 \ \forall n \in \mathbb{N} \ \ n \geqslant N_1 \implies |s_n - s| < \varepsilon$.

For any ε given choose $N_2 = N_1$ (from above), $\forall n \geqslant N_1 = N_2$

$$\big||s_n| - |s|\big| \le |s_n - s| < \varepsilon$$

Hence: $\forall \varepsilon > 0 \ \exists N_2 \ \forall n \in \mathbb{N} \ \ n \geqslant N_2 \implies \big||s_n| - |s|\big| < \varepsilon$

$|s_n| \to |s|$.

9c $\lim s_n = 0 \iff \lim |s_n| = 0$

Proof (i) $\lim s_n = 0 \implies \lim |s_n| = |0| = 0$ by part (a)

Next To prove $\lim |s_n| = 0 \implies \lim s_n = 0$.

Hypothesis: $\lim |s_n| = 0$.

Know $\forall \varepsilon > 0 \ \exists N_1 \ \forall n \in \mathbb{N} \ (n \geqslant N_1 \Rightarrow \big||s_n| - 0\big| < \varepsilon)$

For $\varepsilon > 0$ given choose $N_2 = N_1$ $\forall n \geqslant N_1 = N_2$

$$|s_n - 0| = |s_n| = \big||s_n| - 0\big| < \varepsilon$$

Hence $\forall \varepsilon > 0 \ \exists N_2 \ \forall n \in \mathbb{N} \ \big(n \geqslant N_2 \Rightarrow |s_n - 0| < \varepsilon\big)$

$\lim s_n = 0$.