2.1

Exc 25 bc (pg 51)

(b) $\mathcal{B} = \{ \underbrace{(1, 1+\frac{1}{n})}_{\text{open interval in } \mathbb{R}} \mid n \in \mathbb{N} \}$ A collection of intervals in $\mathbb{R}$.

$(1,2)$

$(1,\frac{3}{2})$

$(1,\frac{4}{3})$

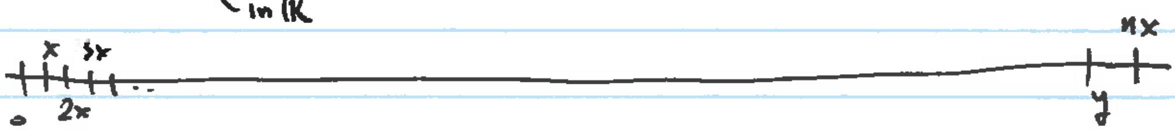
$\boxed{\text{Caution: } B \text{ \& } \mathcal{B} \text{ are different}}$

Let $B_0 = \bigcap_{B \in \mathcal{B}} B = (1,2) \cap (1,\frac{3}{2}) \cap (1,\frac{4}{3}) \cap (1,\frac{5}{4}) \cap \cdots \overset{\text{Claim}}{=} \emptyset$.

see below

We need: Archimedean Principle (old)(classical formulation) 250 BC

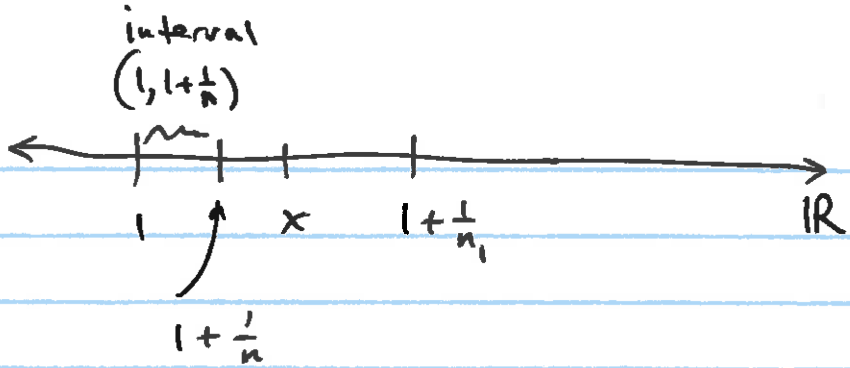
$\forall x,y>0$ (in $\mathbb{R}$) $\exists n \in \mathbb{N}$ s.t. $nx > y$



$1 \notin (1, \frac{1}{n}+1)$ for any $n \in \mathbb{N}$.

$1 \notin B_0$. Suppose $\exists x > 1$ s.t. $x \in B_0$

$x - 1 > 0$, there exists a sufficiently large $n \in \mathbb{N}$ s.t. $\frac{1}{x-1} < n$. (Arch. Principle)



We found n s.t. $\frac{1}{x-1} < n$

$$x - 1 > \frac{1}{n} \quad (\text{since } x-1>0)$$

$$x > 1 + \frac{1}{n}$$

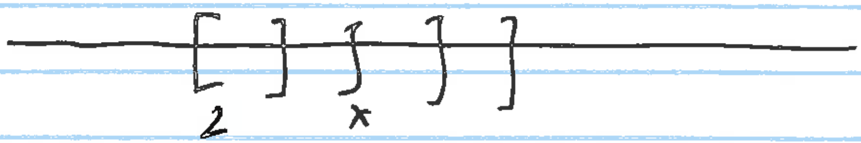
$$x \notin \left(1, 1+\frac{1}{n}\right)$$

Obviously, for any $x \le 1$, $\forall n$ $x \notin (1, 1+\frac{1}{n})$, $x \notin B_0$

Hence $x \notin B_0$, when $x > 1$. $B_0 = \emptyset$.

$$\bigcup_{B\in\mathcal{B}} B = \bigcup_{n=1}^{\infty} \left(1, 1+\frac{1}{n}\right) = (1,2$$

25c) $\mathcal{B} = \{ [2, x] \mid x \in \mathbb{R},\ x > 2 \}$



$$\bigcap_{B\in\mathcal{B}} B = \{2\} \qquad \bigcup_{B\in\mathcal{B}} B = [2, \infty)$$

(we can use
Arch. Principle
for a proper justification
similar to part (b),
to show "if $y > 2$ then $y \notin \bigcap_{B\in\mathcal{B}} B$."

21 True $\forall A, B$

$$A \setminus (A \setminus B) \underset{①}{=} A \cap B = B \setminus (B \setminus A)$$

① To prove $A \setminus (A \setminus B) = A \cap B$ first.

Proof: $x \in A \setminus (A \setminus B) \iff x \in A$ and $x \notin A \setminus B$

$\iff x \in A$ and not $(x \in A \setminus B)$

$\iff x \in A$ and not $(x \in A$ and $x \notin B)$

$\iff x \in A$ and $(x \notin A$ or $x \in B)$

$\iff \underbrace{(x \in A \text{ and } x \notin A)}_{\text{false}}$ or $(x \in A$ and $x \in B)$

$\iff x \in A$ and $x \in B$

$\iff x \in A \cap B$.

We used Logic props. distribution laws

$$\begin{cases} p \wedge (q \vee r) \iff (p \wedge q) \vee (p \wedge r) \\ p \vee (q \wedge r) \iff (p \vee q) \wedge (p \vee r) \end{cases}$$

We proved ① above. $A \setminus (A \setminus B) = A \cap B \quad \forall A, B.$

we can interchange roles

$B \setminus (B \setminus A) = B \cap A.$

$B \cap A = A \cap B$

Hence $A \setminus (A \setminus B) = B \setminus (B \setminus A)$.

2.2 We will do it briefly

Ordered pairs $(a,b) = \{\{a,b\},\{a\}\}$

$\neq$ open interval

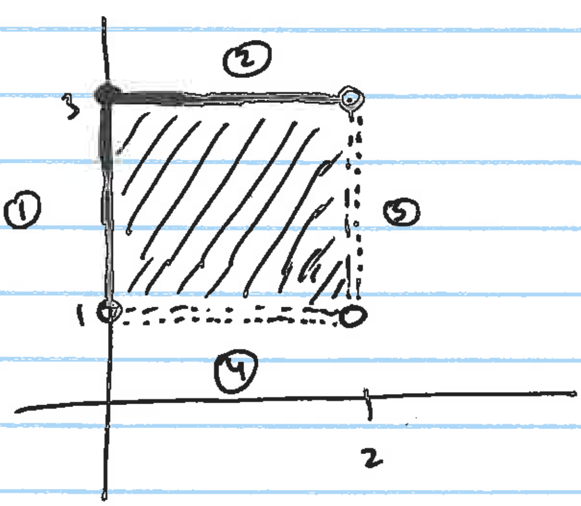
$A \times B = \{(a,b) \mid a \in A, b \in B\}$

Cartesian product.

Ex

$[0,2) \times (1,3] \subseteq \mathbb{R} \times \mathbb{R} = \mathbb{R}^2$

intervals



CAUTION Be careful about confusing notation used in <u>all</u> math literature:

(a,b) means interval in $\mathbb{R}$

(a,b) means an ordered pair in $A \times B$.

You need to make sure to know which one, from the context.