Aug 29, 2018

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1.4

Exc 1.4.11

Prove: For every real # $x>5$, there exist a real number $y<0$ s.t. $x=\frac{5y}{y+3}$.

work sheet scrap not a proof

$$x=\frac{5y}{y+3}$$

$$x(y+3)=5y$$

$$xy+3x=5y$$

$$xy-5y=-3x$$

$$y(x-5)=-3x$$

$$y=\frac{-3x}{x-5}=\frac{3x}{5-x}.$$

Actual proof:

Let $x>5$ be given.

choose $y=\frac{3x}{5-x}$, which is defined in $\mathbb{R}$ since $x>5$.

Since $x>5$ $\quad 3x>0$

$$5-x<0$$

$$y=\frac{3x}{5-x}<0.$$

$$\frac{5y}{y+3}=\frac{5\cdot\frac{3x}{5-x}}{\frac{3x}{5-x}+3}=\frac{(5-x)\,5\cdot\frac{3x}{5-x}}{(5-x)\left(\frac{3x}{5-x}+3\right)}=\frac{15x}{3x+3(5-x)}$$

$5-x\neq 0$

$$=\frac{15x}{3x+15-3x}=\frac{15x}{15}=x.$$

#

Prove

Ex $\forall x \in \mathbb{R} \ (\ |x-2| \le 3 \implies -1 \le x \le 5 \ )$

Proof Let $x \in \mathbb{R}$ such that $|x-2| \le 3$.

There are 2 cases:

Case 1: $x-2 \geq 0$

Case 2: $x-2 < 0$

Case 1 $x-2 \geq 0$

$|x-2| = x-2 \le 3$

$\therefore x \le 5$

$-1 \le 2 \le x$

$-1 \le x \le 5.$

Case 2 $x-2 < 0$

$|x-2| = -(x-2) = 2-x \le 3$

$-1 \le x.$

$x-2 < 0$

$x < 2 \le 5$

$-1 \le x \le 5.$

#

Direct proof of $p \Rightarrow q$

Hypothesis p

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—

— each statement is justified by using previous statements, never using statements afterwards.

—

—

q.

Proof by contradiction for $p \Rightarrow q$.

Assume p

Assume $\sim q$

—

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—

—

contradiction.

Proof by contrapositive for $p \Rightarrow q$

Assume $\sim q$

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—

$\sim p$

Prove

Exple: $\sqrt{2}$ is not a rational number

Recall $\mathbb{Q}$ rational numbers

$$= \left\{ \frac{p}{q} \;\middle|\; p, q \in \mathbb{Z},\ q \neq 0 \right\}.$$

whole numbers

Proof: Proof by contradiction.
Assume the opposite of the conclusion
Suppose $\sqrt{2}$ is a rational number.

$$\sqrt{2} = \frac{p}{q} \quad \text{where } p, q \in \mathbb{Z}.$$

(*) We will assume that p & q have no common factors, by using Prime factorization Thm.

$$(\sqrt{2})^2 = \left(\frac{p}{q}\right)^2$$

$$2 = \frac{p^2}{q^2}$$

$$2q^2 = p^2$$

p^2 is even.

p is even (done it earlier)

Exc. 1.3.7 h

$p = 2k$ for some $k \in \mathbb{Z}$.

$2q^2 = p^2 = (2k)^2 = 4k^2$

$q^2 = 2k^2$

q^2 is even.

q is even.

p & q have a common factor of 2,

Which contradicts $\circledast$ above.

Hence we proved $\sqrt{2} \notin \mathbb{Q}$ by using the "proof by contradiction".

<u>Proving</u> (i) $p \Longrightarrow q$ and r

(ii) $\underbrace{p \Longrightarrow q \text{ or } r}$

actually need to prove
$p \Rightarrow q$
$p \Rightarrow r$

suffices to prove

$(p \text{ and } \sim q) \Longrightarrow r$

OR

$(p \text{ and } \sim r) \Longrightarrow q$

Exc. 1.4.13

If $x^2+x-6 \geq 0$ then $x \leq -3$ or $x \geq 2$.

$\forall x \in \mathbb{R} \left(x^2+x-6 \geq 0 \implies (x \leq -3 \text{ or } x \geq 2) \right)$

It suffices to prove:

$\forall x \in \mathbb{R} (x^2+x-6 \geq 0 \text{ and } x > -3 \implies x \geq 2)$

Proof: Let $x \in \mathbb{R}$ s.t.

Hypothesis $\begin{cases} x^2+x-6 \geq 0, \text{ and} \\ x > -3 \end{cases}$

$$(x^2+x-6) = (x+3)(x-2) \geq 0$$

$$x+3 > 0$$

$$\frac{1}{x+3}(x+3)(x-2) \geq \frac{1}{x+3} \cdot 0$$

$$x-2 \geq 0$$

$$x \geq 2.$$

#.

1.4.23

Exc # 23

not a proof

WS/L: $n, n+2, n+4$

$$n^2 + (n+2)^2 = (n+4)^2$$
$$n^2 + n^2 + 4n + 4 = n^2 + 8n + 16$$
$$2n^2 + 4n + 4 = n^2 + 8n + 16$$
$$n^2 - 4n - 12 = 0$$
$$(n-6)(n+2) = 0$$
$$n = 6 \quad \text{or} \quad n = -2$$

ex $6, 8, 10$ Also $\underbrace{\overset{c}{-10}, \overset{b}{-8}, \overset{a}{-6}}_{\text{but reversed order}}$

$-2, 0, 2$

The Claim "there do not exist three consecutive even integers a, b, c, s.t. $a^2 + b^2 = c^2$" is false.

Counterexample for proof

If we take $a = 6$

$b = 8$

$c = 10$, then $a^2 + b^2 = c^2$

$36 + 64 = 100$.