

1.3

Inductive reasoning

On the basis of looking at individual cases (many) to draw conclusions.

Useful in experimental sciences.

but Not in Math

Ex (a) $\forall n \in \mathbb{N}$, $n^2 + n + 11$ is prime: TRUE OR FALSE?

1	13
2	17
3	23
4	31
5	41
6	53
7	67
8	83
9	101
10	121

prime for the first 9 tries.

statement is false

not prime

In Math we use deductive reasoning:

Applying a general principle to a particular case.

Exc 1.3.1

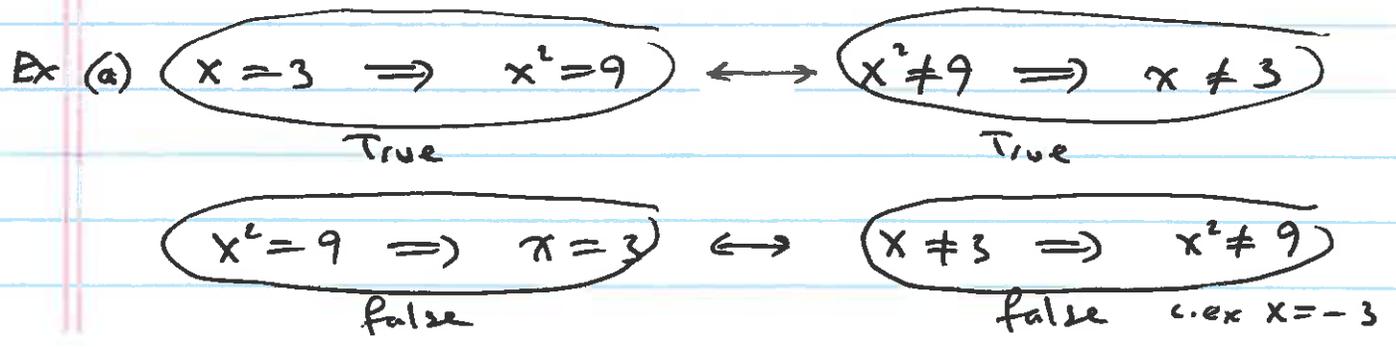
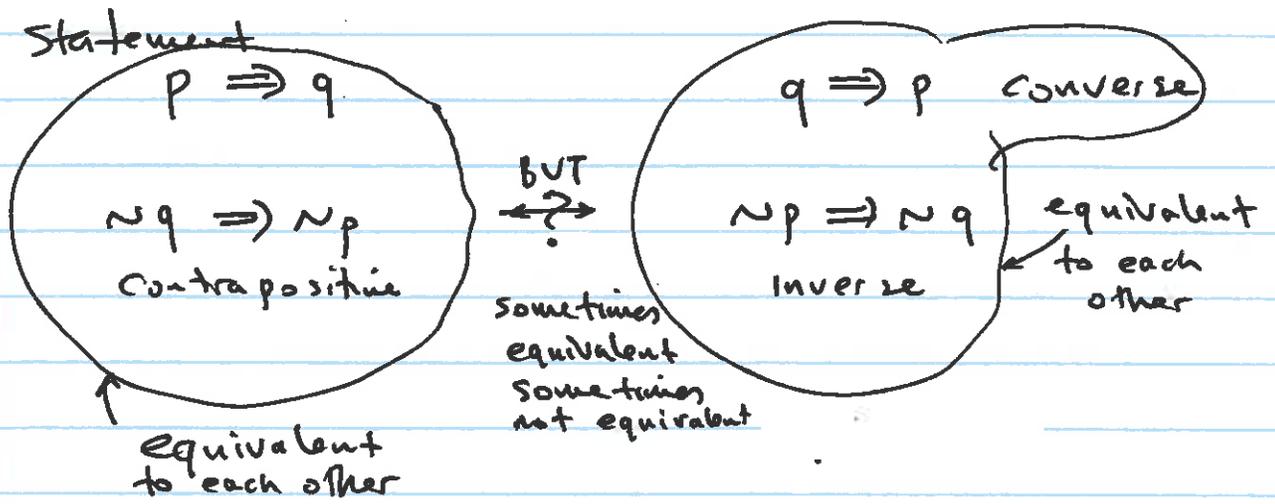
FALSE d) To prove $\forall n p(n)$ is true it takes one example
 TRUE e) To prove $\exists n p(n)$ is true it takes one example

1.3.2

TRUE d) To prove $\forall n p(n)$ is false it takes one counterex.
 FALSE e) To prove $\exists n p(n)$ is false it takes one counterex.

Ex i) $\forall x \in \mathbb{R} \quad x^2 > 0$ false since $\exists x=0 \in \mathbb{R}$
 $0^2 > 0$ is false

ii) $\exists x \in \mathbb{R} \quad x > 1$ true since $\exists x=2 > 1$.



3.3c

f continuous and C is connected $\Rightarrow f(C)$ is connected

Contrapositive:

$f(C)$ is not connected \Rightarrow f is not continuous
OR
 C is not connected.

3.6 Find counterexamples:

a) $\forall x \in \mathbb{R} \quad x^2 > 9 \Rightarrow x > 3$

$x = -4$ is a counterexample

$(-4)^2 = 16 > 9$ Hypothesis \checkmark

$-4 > 3$ is false Conclusion is false.

(u) No rational number x satisfies

$$x^3 + (x-1)^2 = x^2 + 1.$$

Work: $x^3 + x^2 - 2x + 1 = x^2 + 1$

$$x^3 - 2x = 0$$

$$x(x^2 - 2) = 0$$

$x = 0$ is the only counterex.

$$x = \pm\sqrt{2}$$

3.7 abfh

Integers $\mathbb{Z} = \{0, 1, -1, 2, -2, 3, -3, \dots\}$
 m is even if $m = 2k$ for some $k \in \mathbb{Z}$

m is odd if $m = 2k+1$ " " "

(a) $(p \text{ is odd} \wedge q \text{ is odd} \Rightarrow p+q \text{ is even.})$

Hidden quantifier $\forall p, q \in \mathbb{Z}$

Proof: p is odd and q is odd.
 p is odd
 $p = 2k+1$ for some $k \in \mathbb{Z}$.
 q is odd.
 $q = 2j+1$ for some $j \in \mathbb{Z}$.

$$\begin{aligned} p+q &= (2k+1) + (2j+1) \\ &= 2k + 2j + 2 \\ &= 2(k+j+1) \end{aligned}$$

$$k, j \in \mathbb{Z} \Rightarrow k+j+1 \in \mathbb{Z}.$$

$$p+q = 2 \cdot m \quad \text{where } m = k+j+1 \in \mathbb{Z}.$$

$$p+q \text{ is even.}$$

QED.

(5)

(b) $\forall p, q \in \mathbb{Z} (p \text{ is odd} \wedge q \text{ is odd} \Rightarrow pq \text{ is odd.})$

Proof:

p is odd \wedge q is odd.

p is odd

$p = 2k + 1$ for some $k \in \mathbb{Z}$.

q is odd

$q = 2j + 1$ for some $j \in \mathbb{Z}$.

$$\begin{aligned} p \cdot q &= (2k + 1)(2j + 1) \\ &= 4kj + 2k + 2j + 1 \\ &= 2(2kj + k + j) + 1 \end{aligned}$$

$m = 2kj + k + j$ is an integer

$$p \cdot q = 2m + 1$$

$p \cdot q$ is odd.

QED

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(h) If p^2 is even, then p is even. \otimes

Direct proof involves Prime factorization Thm.

We will try to prove the contrapositive:

$$p \text{ is not even} \Rightarrow p^2 \text{ is not even}$$

Need: $(p \text{ is not even} \Leftrightarrow p \text{ is odd})$

WTS: $p \text{ is odd} \Rightarrow p^2 \text{ is odd}$.

We actually proved $\text{in (b)} \quad p \text{ is odd} + q \text{ is odd} \Rightarrow pq \text{ is odd}$.

By taking $p=q$

$p \text{ is odd} \Rightarrow p^2 \text{ is odd}$ is a consequence of (b) which we proved.

Hence \otimes is true, since its contrapositive is true. #

Given an integer n , We divide n by 2. If n is divisible by 2, then n is even. If the remainder is 1, then n is odd. There is no third possibility. Hence every integer is either odd or even, and not both.