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Which ones are true?

T $2=3 \Rightarrow 5=5$

T $2=3 \Rightarrow 5=6$

T $2=2 \Rightarrow 5=5$

F $2=2 \Rightarrow 5=6$

→ Can you prove $2=3 \Rightarrow 5=5$?

$2=3$

$\Downarrow$

$5=5$

Proof: $2=3$ hypothesis

$3=2$ (change Left to right)

$2+3=3+2$ (added LHS & added RHS)

$5=5$ Conclusion.

→ Can you prove $2=3 \Rightarrow 5=6$?

Proof: $2=3$

$3=3$

$2+3=3+3$

$5=6$

Ex Bertrand Russel 1901 gave a proof of "$1=2 \Rightarrow$ BR is the Pope."

Ex $x^2=4 \Rightarrow x=2$?

Is it true?

(i) If $x \in \mathbb{R}$, then $x^2=4 \Rightarrow x=2$ is false

if $x \in \mathbb{N}$, then $x^2=4 \Rightarrow x=2$ is true

(ii) In
$x = 2 \Rightarrow x^2 = 4$,
it means that if you plug in $x=2$, then $x^2=4$.

In $x^2=4 \Rightarrow x=2$, you are claiming that
the equation $x^2=4$ has the only sol^n $x=2$.
In Natural numbers $\mathbb{N}$, this is true
In Real numbers $\mathbb{R}$, it is false since also
$x=-2$ is a possible solution

Discuss: $\Longleftrightarrow$

Defn $p \Longleftrightarrow q$ is equivalent to
$$((p \Rightarrow q) \text{ and } (q \Rightarrow p))$$

$\Longleftrightarrow$
if and only if
iff
$\equiv$
} commonly used terminology

De Morgan's Laws:

$$\sim(p \text{ and } q) \Longleftrightarrow ((\sim p) \text{ or } (\sim q))$$
$$\sim(p \text{ or } q) \Longleftrightarrow ((\sim p) \text{ and } (\sim q))$$

Consequently:

$$\sim(p \Rightarrow q) \Longleftrightarrow ?$$ PTO

Two groups are **not** necessarily equivalent

$$\left.\begin{array}{ll} p \Rightarrow q & \text{statement} \\ \sim q \Rightarrow \sim p & \text{contrapositive} \\ \sim p \text{ OR } q & \end{array}\right\} \text{logically equivalent}$$

$$\left.\begin{array}{ll} q \Rightarrow p & \text{converse} \\ \sim p \Rightarrow \sim q & \text{inverse} \end{array}\right\} \text{equivalent}$$

Since We know: $p \Rightarrow q$ equivalent $\sim p$ OR q.

$\text{not}\,(p \Rightarrow q)$ equivalent $\sim(\sim p \text{ or } q)$

equivalent p and $\sim q$.

counterexample to $p \Rightarrow q$.

<u>Example</u> I live in Iowa $\Rightarrow$ I live in USA. (True)

Converse?

$$\underbrace{\text{I live in USA} \Rightarrow \text{I live in Iowa.}}_{\text{false}}$$

Why? A counterexample?

I live in Illinois (hence in USA) Hypothesis ✓

but not in Iowa. Conclusion X.

Ex Find counterexample:

If $x \in \mathbb{R}$: $x^2 = 4 \Rightarrow x = 2$.

Counterexample: $x = -2$ satisfies hypothesis $x^2 = 4$
but not the conclusion $x = 2$.

Exercises

p9 #2 a) F p is the hypothesis, or antecedent

b) T

c) F (If p, then q) $\Longleftrightarrow$ (q whenever p)

d) T De Morgan's Laws

e) F $q \Rightarrow p$ is the converse of $p \Rightarrow q$.

In general the converse may or may not have the same validity.

Ex In real numbers

		converse
F	$x^2 = 4 \Rightarrow x = 2$	T $x = 2 \Rightarrow x^2 = 4$
T	$x^2 = 0 \Rightarrow x = 0$	T $x = 0 \Rightarrow x^2 = 0$

p9 #9

a) T (since T and T)

b) T (since $\underbrace{F \text{ or } T}_{T}$)

c) F (since $\underbrace{F \text{ or } F}_{F}$)

d) T (since $T \Rightarrow T$)

e) T (since $\underbrace{F \Rightarrow F}_{T}$)

f) F (since $\underbrace{T \Rightarrow F}_{F}$)

g) T (since $(\underbrace{T \text{ and } F}_{F}) \Rightarrow \text{True}$)

h) F (since $\underbrace{(\underbrace{T \text{ or } F}_{T}) \Rightarrow F}_{F}$)

i) T (since $\underbrace{\underbrace{T \text{ and } F}_{F} \Rightarrow F}_{T}$)

j) Caution: Depends on ( ) or where comma "," is.

False   It is not the case that $(\underbrace{\underbrace{\text{5 is even}}_{F} \text{ or } \underbrace{\text{7 is prime}}_{T}}_{T})$

True $(\underbrace{\text{It is not the case that 5 is even}}_{T})$ OR $\underbrace{\text{7 is prime}}_{T}$