Feb 27, 2020

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Exam 3/2/2020 Monday 6:30 - 8:30

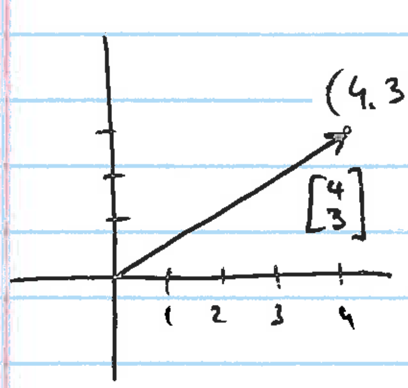
W 151 PBB

for 0331 section.

2.9 Recall Basis $\mathcal{B}$ for a subspace H:

- $\mathcal{B}$ Spans H, and
- $\mathcal{B}$ is linearly independent

ex. 1



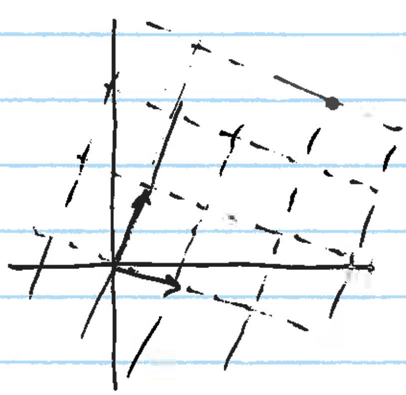
$$\begin{bmatrix}4\\3\end{bmatrix} = 4\begin{bmatrix}1\\0\end{bmatrix} + 3\begin{bmatrix}0\\1\end{bmatrix}$$

Standard basis for $\mathbb{R}^2$

$$\left\{\begin{bmatrix}1\\0\end{bmatrix}, \begin{bmatrix}0\\1\end{bmatrix}\right\}$$

ex 2

$$\mathcal{B} = \left\{\begin{bmatrix}1\\2\end{bmatrix}, \begin{bmatrix}2\\-1\end{bmatrix}\right\} \text{ basis for } \mathbb{R}^2$$



$$3\begin{bmatrix}1\\2\end{bmatrix} + 2\begin{bmatrix}2\\-1\end{bmatrix} = \begin{bmatrix}3+4\\6-2\end{bmatrix}$$

$$= \begin{bmatrix}7\\4\end{bmatrix}.$$

For the point $\overset{P}{\overset{\shortparallel}{\begin{bmatrix}7\\4\end{bmatrix}}} = 7\begin{bmatrix}1\\0\end{bmatrix} + 4\begin{bmatrix}0\\1\end{bmatrix}$

coordinates wrt $\left\{\begin{bmatrix}1\\0\end{bmatrix}, \begin{bmatrix}0\\1\end{bmatrix}\right\}$

$$\begin{bmatrix}7\\4\end{bmatrix} = 3\begin{bmatrix}1\\2\end{bmatrix} + 2\begin{bmatrix}2\\-1\end{bmatrix}$$

coordinates wrt $\left\{\begin{bmatrix}1\\2\end{bmatrix}, \begin{bmatrix}2\\-1\end{bmatrix}\right\}$

Defn Given a basis $\mathcal{B} = \{\vec{b}_1, \vec{b}_2, \dots \vec{b}_p\}$ of a subspace H, and a point $q \in H$, s.t.

$$q = c_1\vec{b}_1 + c_2\vec{b}_2 + \dots + c_p\vec{b}_p$$

for some $c_1, c_2, \dots c_p$,

then

$$[q]_{\mathcal{B}} = \begin{bmatrix}c_1\\c_2\\\vdots\\c_p\end{bmatrix},$$

called coordinates of q with respect to the basis $\mathcal{B} = \{\vec{b}_1, \vec{b}_2, \dots \vec{b}_p\}$

Obs There is only one way to write

$$q = c_1\vec{b}_1 + \cdots + c_p\vec{b}_p \quad \text{wrt} \quad \{\vec{b}_1, \vec{b}_2, \ldots \vec{b}_p\}.$$

Suppose

$$q = \underbrace{c_1\vec{b}_1 + \cdots c_p\vec{b}_p} = \underbrace{c_1'\vec{b}_1 + c_2'\vec{b}_2 + \cdots} + c_p'\vec{b}_p$$

$$(c_1 - c_1')\vec{b}_1 + (c_2 - c_2')\vec{b}_2 + \cdots + (c_p - c_p')\vec{b}_p = 0$$

$\{b_1, \ldots b_p\}$ lin indep dent $\Rightarrow$

$$c_1 - c_1' = 0$$
$$c_2 - c_2' = 0$$
$$\vdots$$
$$c_p - c_p' = 0$$

$$\Rightarrow \left.\begin{array}{l} c_1 = c_1' \\ c_2 = c_2' \\ \vdots \\ c_p = c_p' \end{array}\right\} \text{Only one way}$$

Exc #6

$$b_1 = \begin{bmatrix} -3 \\ 1 \\ -4 \end{bmatrix}, \quad b_2 = \begin{bmatrix} 7 \\ 5 \\ -6 \end{bmatrix} \qquad x = \begin{bmatrix} 11 \\ 0 \\ 7 \end{bmatrix}$$

Question is to find c_1, c_2 s.t.

$$c_1 \begin{bmatrix} -3 \\ 1 \\ -4 \end{bmatrix} + c_2 \begin{bmatrix} 7 \\ 5 \\ -6 \end{bmatrix} = \begin{bmatrix} 11 \\ 0 \\ 7 \end{bmatrix}$$

$$\left[\begin{array}{cc|c} -3 & 7 & 11 \\ 1 & 5 & 0 \\ -4 & -6 & 7 \end{array}\right] \xrightarrow[3R_2 + R_1 \to R_1]{} \left[\begin{array}{cc|c} 0 & 22 & 11 \\ 1 & 5 & 0 \\ -4 & -6 & 7 \end{array}\right]$$

$$\xrightarrow[4R_2 + R_3 \to R_3]{} \left[\begin{array}{cc|c} 0 & 22 & 11 \\ 1 & 5 & 0 \\ 0 & 14 & 7 \end{array}\right] \xrightarrow[\frac{1}{22}R_1,\ \frac{1}{14}R_3]{} \left[\begin{array}{cc|c} 0 & 1 & \frac{1}{2} \\ 1 & 5 & 0 \\ 0 & 1 & \frac{1}{2} \end{array}\right]$$

$$\xrightarrow[R_1 \Leftrightarrow R_2]{} \left[\begin{array}{cc|c} 1 & 5 & 0 \\ 0 & 1 & \frac{1}{2} \\ 0 & 1 & \frac{1}{2} \end{array}\right] \longrightarrow \left[\begin{array}{cc|c} 1 & 0 & -\frac{5}{2} \\ 0 & 1 & \frac{1}{2} \\ 0 & 0 & 0 \end{array}\right]$$

$$-\frac{5}{2} \begin{bmatrix} -3 \\ 1 \\ -4 \end{bmatrix} + \frac{1}{2} \begin{bmatrix} 7 \\ 5 \\ -6 \end{bmatrix} = \begin{bmatrix} 11 \\ 0 \\ 7 \end{bmatrix}$$

$$[x]_{\mathcal{B}} = \begin{bmatrix} -\frac{5}{2} \\ \frac{1}{2} \end{bmatrix} \quad \text{where} \quad \mathcal{B} = \{\vec{b}_1, \vec{b}_2\}$$

Prop Given a linear subspace H with two bases for H:

$$\mathcal{B}_1 = \{\vec{b}_1, \vec{b}_2, \ldots \vec{b}_p\}$$

$$\mathcal{B}_2 = \{\vec{a}_1, \vec{a}_2, \ldots \vec{a}_q\}.$$

Then one has $p = q$,
That is these bases have the same number of vectors.

Defn Dimension of H = # vectors in every (any) basis of H.
if $H \neq \{\vec{0}\}$.

If $H = \{\vec{0}\}$, $\dim H = 0$.

Ex

$$\dim \operatorname{span} \left\{ \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \right\} = 2$$

dependent on $\begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}$.

$$\dim \operatorname{span} \left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \end{bmatrix} \right\}$$

$$= \dim \operatorname{span} \{ \underbrace{\begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}}_{\text{lin indep}} \} = 2$$

2.9 Exc #10

$$\dim \text{col space} = 3$$
$$+$$
$$\dim \text{nul space} = 2$$
$$\|$$
$$\left.\begin{array}{l}\#\text{ unknowns}\\ \#\text{ columns}\end{array}\right\} = 5$$

$$A = \begin{bmatrix} 1 & -2 & 9 & 5 & 4 \\ 1 & -1 & 6 & 5 & -3 \\ -2 & 0 & -6 & 1 & -2 \\ 4 & 1 & 9 & 1 & -4 \end{bmatrix}$$

$$\downarrow$$

$$\begin{bmatrix} 1 & -2 & 9 & 5 & 4 \\ 0 & 1 & -3 & 0 & -7 \\ 0 & 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\xrightarrow{2R_2+R_1} \begin{bmatrix} 1 & 0 & 3 & 5 & -10 \\ 0 & 1 & -3 & 0 & -7 \\ 0 & 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \xrightarrow{-5R_3+R_1 \to R_1} \underbrace{\begin{bmatrix} 1 & 0 & 3 & 0 & 0 \\ 0 & 1 & -3 & 0 & -7 \\ 0 & 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}}_{\text{pivots}}$$

$$\text{Col space} = \text{span}\left\{ \begin{bmatrix} 1 \\ 1 \\ -2 \\ 4 \end{bmatrix}, \begin{bmatrix} -2 \\ -1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 5 \\ 5 \\ 1 \\ 1 \end{bmatrix} \right\}$$

$$\dim \text{col space} = 3 = \#\text{ of pivots of RREF of } A$$

Next solve $Ax = 0$

$$x_1 + 3x_3 = 0$$
$$x_2 - 3x_3 - 7x_5 = 0$$
$$x_4 - 2x_5 = 0$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = x_3 \begin{bmatrix} -3 \\ 3 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_5 \begin{bmatrix} 0 \\ 7 \\ 0 \\ 2 \\ 1 \end{bmatrix}$$

$$\text{Nul } A = \text{Null space} = \text{Span}\left\{ \begin{bmatrix} -3 \\ 3 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 7 \\ 0 \\ 2 \\ 1 \end{bmatrix} \right\}$$

$$\dim \text{nul space} = 2 \ (\#\text{ of parameters})$$