Feb 6, 2020

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Defn A SLE is called homogenous if it is in the form

$$A\cdot\vec{x} = \vec{0}$$

Otherwise, it is called non-homogenous.

Ex

all zeros for constants

$$\left.\begin{aligned} x + 2y &= 0 \\ x - y &= 0 \end{aligned}\right\} \text{homogenous}$$

$$\left.\begin{aligned} x_1 + 3x_2 - x_3 &= 4 \\ x_1 \qquad + x_3 &= 2 \\ x_1 + x_2 - x_3 &= 0 \end{aligned}\right\} \text{not homogenous}$$

Defn The trivial solution: $\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$

Prop: • Every homogenous system of LE is consistent since, the trivial solution is a solution of that system.

• If an SLE has a trivial solution then it is of the form $A\cdot\vec{x} = \vec{0}$, that is, it is homogenous.

Q: I am given a homogenous SLE.
I know trivial solution is good.
Are there others?

Ex
$$\left.\begin{aligned} x \quad + z &= 0 \\ 2x - y + z &= 0 \\ 5x - 2y + 3z &= 0 \end{aligned}\right\}$$

of course $x = y = z = 0$ is $\underline{\underline{a}}$ solution.

(HW check!)

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 2 & -1 & 1 & 0 \\ 5 & -2 & 3 & 0 \end{array}\right] \dashrightarrow \left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & -1 & -1 & 0 \\ 0 & -2 & -2 & 0 \end{array}\right]$$

$$\dashrightarrow \underbrace{\left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]}_{\text{RREF}}$$

pivots

free

$x + z = 0$
$y + z = 0$
$z = \text{free}$

$$\underbrace{\begin{aligned} x &= -z \\ y &= -z \\ z &= z \text{ free} \end{aligned}}$$

scalar parametric representation

free

$$\underbrace{\begin{bmatrix} x \\ y \\ z \end{bmatrix} = z \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}}$$

parametric vector representation/form

For $\begin{cases} x \quad\quad + z = 0 \\ 2x - y + z = 0 \\ 5x - 2y + 3z = 0 \end{cases}$

Solution space $= \text{Span}\left(\begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix} \right)$

Ex given SLE

$$\begin{array}{ccccc} x_1 & x_2 & x_3 & x_4 & \\ \end{array}$$

$$\left[\begin{array}{cccc|c} 1 & -3 & -8 & 5 & 0 \\ 0 & 1 & 2 & -4 & 0 \end{array}\right] \leftarrow \text{homogenous}$$

$$\downarrow RR$$

$$\left[\begin{array}{cccc|c} 1 & 0 & -2 & -7 & 0 \\ 0 & 1 & 2 & -4 & 0 \end{array}\right]$$

pivot free variables

$x_1 - 2x_3 - 7x_4 = 0$

$x_2 + 2x_3 - 4x_4 = 0$

$x_1 = 2x_3 + 7x_4$

$x_2 = -2x_3 + 4x_4$

$x_3 =$ free (x_3) (no x_4)

$x_4 =$ free (x_4) (no x_3)

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \underbrace{x_3 \begin{bmatrix} 2 \\ -2 \\ 1 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} 7 \\ 4 \\ 0 \\ 1 \end{bmatrix}}_{\text{linear combination}}$$

Vector parametric sol'n

x_3, x_4 free

Solution space $= \text{Span}\left(\begin{bmatrix} 2 \\ -2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 7 \\ 4 \\ 0 \\ 1 \end{bmatrix} \right).$

Thm: A homogenous SLE $A\cdot\vec{x}=\vec{0}$ has a non-trivial solution

$\iff$ The RREF of A admits a free variable i.e. RREF of A has a column without a leading 1.

Summarize Given $A\cdot\vec{x}=\vec{0}$

① If every column of RREF of A has a leading 1, then $\vec{x}=\vec{0}$ is the only sol^n.

② The number of the parameters is the # of the columns of RREF(A) without a leading 1.

③ In all cases, the solution set of $A\cdot\vec{x}=\vec{0}$ is the span of finitely many vectors.

Non-Homogenous SLE:

- May or May NOT have solutions.

Ex

$$2x+y+z=3$$
$$x+y-z=2$$

$$\left[\begin{array}{ccc|c} 2 & 1 & 1 & 3 \\ 1 & 1 & -1 & 2 \end{array}\right] \xrightarrow{RR} \left[\begin{array}{ccc|c} 1 & 0 & 2 & 1 \\ 0 & 1 & -3 & 1 \end{array}\right].$$

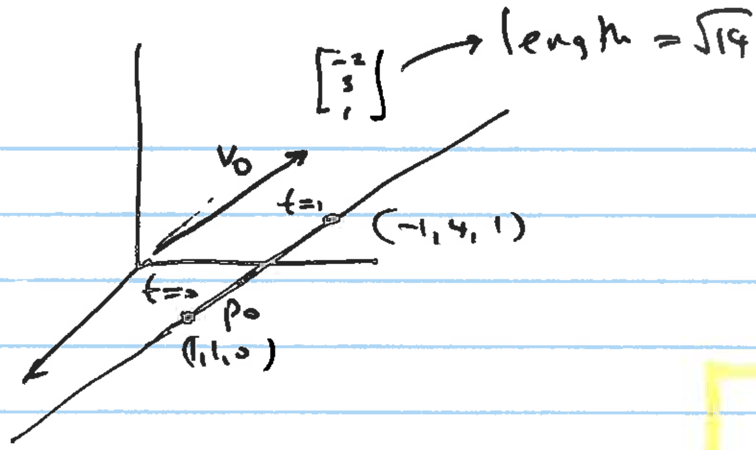
pivots   free

$$x = 1-2z$$
$$y = 1+3z$$
$$z = \text{free } z$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \underbrace{\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + z\begin{bmatrix} -2 \\ 3 \\ 1 \end{bmatrix}}$$

line passing through $\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$

& parallel to $\begin{bmatrix} -2 \\ 3 \\ 1 \end{bmatrix}$.



$p_0 + z v_0$

usually we write

$$\boxed{\underbrace{\vec{p}_0 + t\vec{v}_0}_{\text{line through } p_0 \text{ parallel to } v_0}}$$

Compare :

$\underbrace{\begin{aligned} 2x + y + z &= 3 \\ x + y - z &= 2 \end{aligned}}$	$\begin{aligned} 2x + y + z &= 0 \\ x + y - z &= 0 \end{aligned}$
Sol^n $\begin{bmatrix}1\\1\\0\end{bmatrix} + z\begin{bmatrix}-2\\3\\1\end{bmatrix}$	Sol^n $z\begin{bmatrix}-2\\3\\1\end{bmatrix}$

Thm: Let $A\vec{x} = \vec{b}$ be given.

- If p_0 is a (particular) solution: $A \cdot \vec{p}_0 = b$,

  and

- V_h describes a complete parametric solution of the homogenous eq^n $A \cdot \vec{x} = \vec{0}$

Then the complete solution of $A \cdot \vec{x} = b$ is of the form

$$\vec{p}_0 + \vec{V}_h$$

~ #2 parametric equation of a line

· passing thru $\begin{bmatrix} -3 \\ 2 \end{bmatrix}$

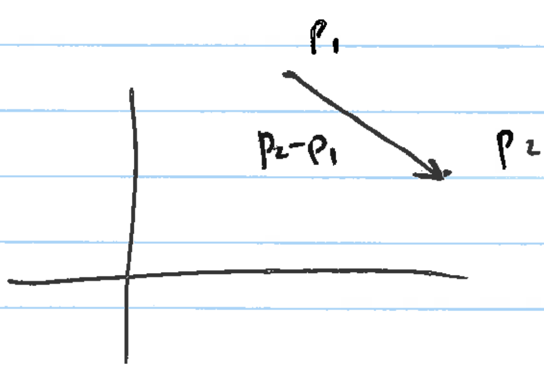
· parallel to $\begin{bmatrix} -7 \\ 6 \end{bmatrix}$.

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -3 \\ 2 \end{bmatrix} + t \begin{bmatrix} -7 \\ 6 \end{bmatrix}$$

~ #2 param eq^n of a line thru $\begin{bmatrix} -3 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ -3 \end{bmatrix}$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \underbrace{\begin{bmatrix} -3 \\ 2 \end{bmatrix}}_{\text{thru}} + t \underbrace{\begin{bmatrix} 3 \\ -5 \end{bmatrix}}_{\text{parallel}}$$

$$\begin{bmatrix} 0 \\ -3 \end{bmatrix} - \begin{bmatrix} -3 \\ 2 \end{bmatrix} = \begin{bmatrix} 3 \\ -5 \end{bmatrix}$$



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