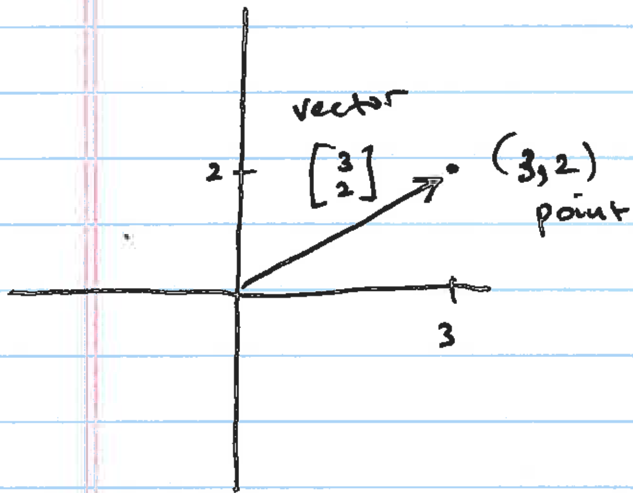
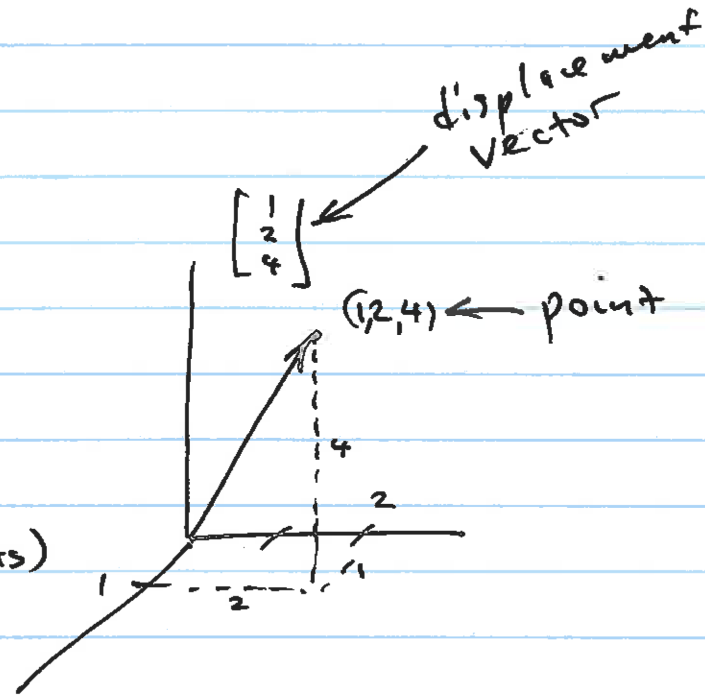
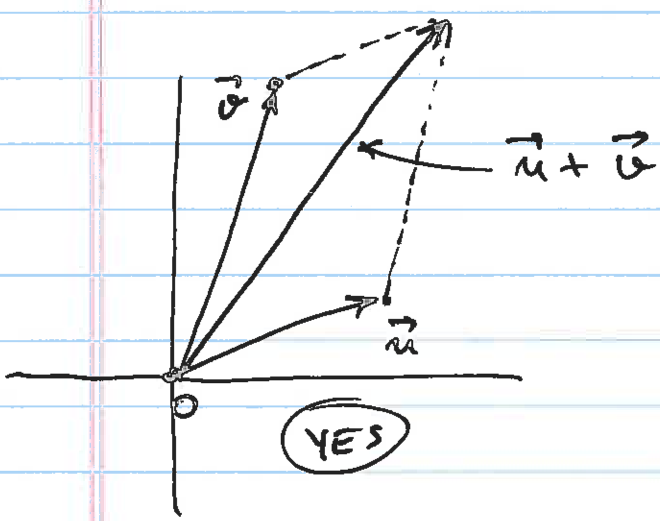
1.3 Continue

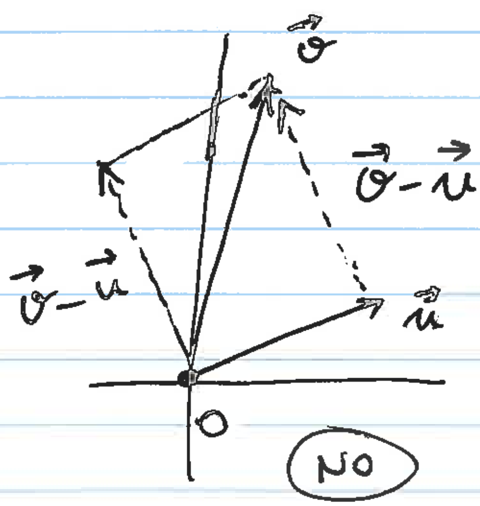




You Can add vectors (displacements)
You Cannot add points.

Parallelogram Law





If one places a parallelogram whose 3 vertices are the terminal points of the vectors $\vec{u}, \vec{0}, \vec{v}$ s.t. the terminal pts of $\vec{u}$ and $\vec{v}$ are not adjacent on the parallelogram then the 4^{th} vertex is the terminal pt of $\vec{u} + \vec{v}$

Linear Combinations.

<u>Defn</u> Given vectors $\vec{v}_1, \vec{v}_2, \dots \vec{v}_k$ in $\mathbb{R}^n$ & real numbers $c_1, c_2, \dots c_k$ one defines

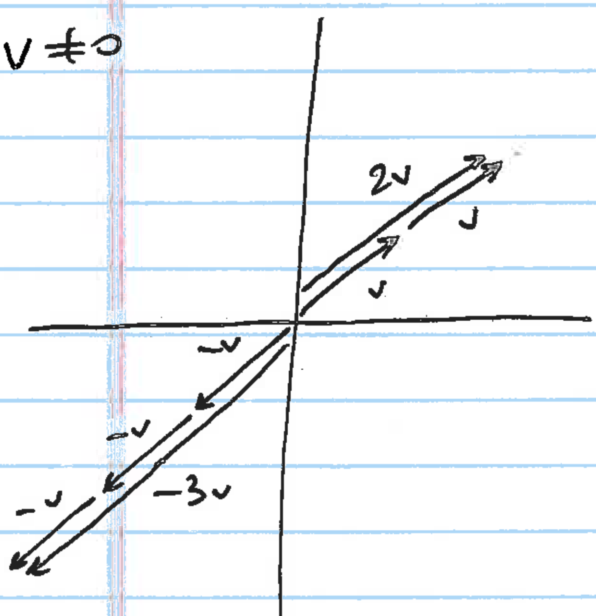
$$c_1\vec{v}_1 + c_2\vec{v}_2 + c_3\vec{v}_3 + \dots + c_k\vec{v}_k$$ to be the linear combination of vectors $\vec{v}_1, \dots \vec{v}_k$ with weights $c_1, c_2, \dots c_k$.

Geometrically

Fix $v \in \mathbb{R}^n$ $\{c\vec{v} \mid c \in \mathbb{R}\}$

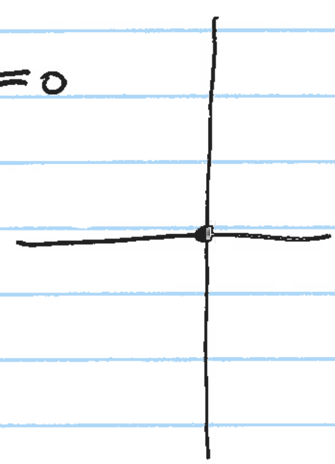
The collection of all $c\vec{v}$ (all possible linear combinations using v, as c runs thru $\mathbb{R}$

$v \neq 0$

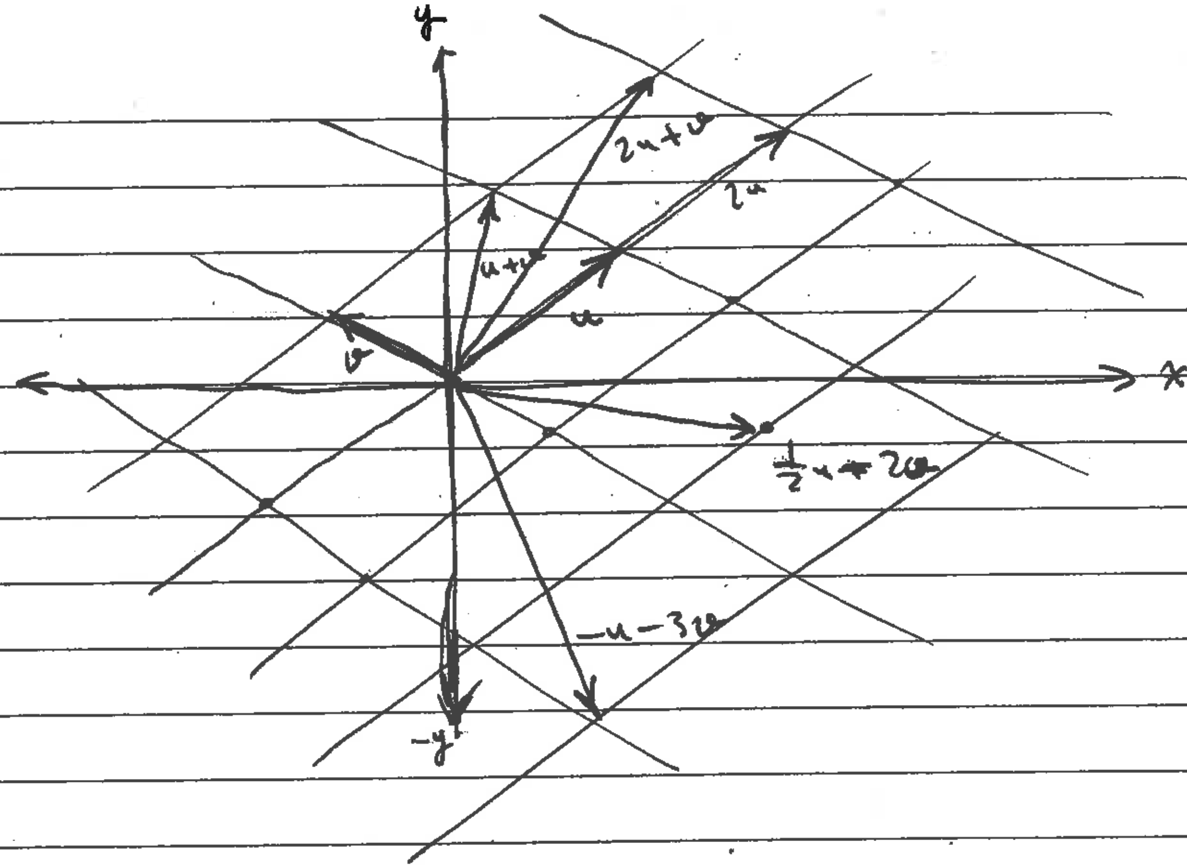


The tips of the vectors $c\vec{v}$, are on a line passing thru origin, & parallel to v.

$\vec{v} = 0$



for all $\forall c \in \mathbb{R}$ $cv = 0$



The linear combinations of the form

$c_1\vec{u} + c_2\vec{v}$ (where $u \neq 0$, $v \neq 0$, u is not parallel to v)

with some chosen values of c_1 and c_2.

Ex: Linear Combinations

$$\underbrace{2\begin{bmatrix}1\\-1\\6\end{bmatrix}+4\begin{bmatrix}1\\-1\\3\end{bmatrix}+(-2)\begin{bmatrix}1\\1\\0\end{bmatrix}}_{\text{linear combination}}=\begin{bmatrix}4\\-8\\24\end{bmatrix}$$

Question: Is $\begin{bmatrix}5\\-1\\4\end{bmatrix}$ a linear combination of $\begin{bmatrix}1\\1\\2\end{bmatrix}, \begin{bmatrix}-1\\1\\0\end{bmatrix}$?

which is asking:

Does there exist $x_1, x_2 \in \mathbb{R}$ s.t.

(*) $$\begin{bmatrix}5\\-1\\4\end{bmatrix}=x_1\begin{bmatrix}1\\1\\2\end{bmatrix}+x_2\begin{bmatrix}-1\\1\\0\end{bmatrix} \quad ?$$

$$\begin{bmatrix}5\\-1\\4\end{bmatrix}=\begin{bmatrix}x_1-x_2\\x_1+x_2\\2x_1+0x_2\end{bmatrix}$$

(**) $$\left.\begin{aligned} x_1-x_2&=5\\ x_1+x_2&=-1\\ 2x_1\quad&=4\end{aligned}\right\}\text{SLE}$$

$$\left[\begin{array}{cc:c}1&-1&5\\1&1&-1\\2&0&4\end{array}\right].$$

The original question becomes "Is it consistent?"

Reduce

$$\left[\begin{array}{cc|c} 1 & -1 & 5 \\ 1 & 1 & -1 \\ 2 & 0 & 4 \end{array}\right] \xrightarrow[R_2-R_1 \,\to\, R_2]{} \left[\begin{array}{cc|c} 1 & -1 & 5 \\ 0 & 2 & -6 \\ 2 & 0 & 4 \end{array}\right]$$

$$\xrightarrow[\frac{1}{2}R_3]{\frac{1}{2}R_2} \left[\begin{array}{cc|c} 1 & -1 & 5 \\ 0 & 1 & -3 \\ 1 & 0 & 2 \end{array}\right] \xrightarrow[R_1 \leftrightarrow R_3]{} \left[\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & -3 \\ 1 & -1 & 5 \end{array}\right]$$

$$\xrightarrow[R_3-R_1 \,\to\, R_3]{} \left[\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & -3 \\ 0 & -1 & 3 \end{array}\right] \xrightarrow[R_2+R_3 \,\to\, R_3]{} \left[\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & -3 \\ 0 & 0 & 0 \end{array}\right]$$

$x_1 = 2$
$x_2 = -3$

Put pack into the eq^n (*) p.④:

$$\begin{bmatrix} 5 \\ -1 \\ 4 \end{bmatrix} = 2\begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} - 3\begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}.$$

So consistency of (**) tells us that

$\begin{bmatrix} 5 \\ -1 \\ 4 \end{bmatrix}$ is a linear combination of $\begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$.

Exc.

1.3 # 14 (Similar)

$$A = \begin{bmatrix} 1 & 0 & 5 \\ -2 & 1 & -6 \\ 0 & 2 & 8 \end{bmatrix} \qquad b = \begin{bmatrix} 2 \\ -1 \\ 6 \end{bmatrix}$$

Is b a linear combination of the column vectors of A? YES Why?

Asking: Are there any x_1, x_2, x_3 s.t.

$$x_1 \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} + x_3 \begin{bmatrix} 5 \\ -6 \\ 8 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 6 \end{bmatrix}$$

SLE

$$\left.\begin{aligned} x_1 + 0x_2 + 5x_3 &= 2 \\ -2x_1 + x_2 - 6x_3 &= -1 \\ 0x_1 + 2x_2 + 8x_3 &= 6 \end{aligned}\right\} (*)$$

****

$\boxed{\text{Compare to } A, b \text{ above}}$

Matrix of SLE

$$\left[\begin{array}{ccc|c} 1 & 0 & 5 & 2 \\ -2 & 1 & -6 & -1 \\ 0 & 2 & 8 & 6 \end{array}\right] \xrightarrow[\substack{\downarrow \\ R_2}]{2R_1 + R_2} \left[\begin{array}{ccc|c} 1 & 0 & 5 & 2 \\ 0 & 1 & 4 & 3 \\ 0 & 2 & 8 & 6 \end{array}\right]$$

$$\xrightarrow[\substack{\downarrow \\ R_3}]{R_3 - 2R_2} \left[\begin{array}{ccc|c} 1 & 0 & 5 & 2 \\ 0 & 1 & 4 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

pivot

Free

$$\underbrace{\begin{aligned} x_1 &= 2 - 5x_3 \\ x_2 &= 3 - 4x_3 \\ x_3 &= \text{free} \end{aligned}}$$

we can find infinitely many different x_1, x_2, x_3 s.t. $(*)$ holds.

1.3 END