

①.2 Row Reduced Echelon Form Matrix RREF: RREF:

Echelon

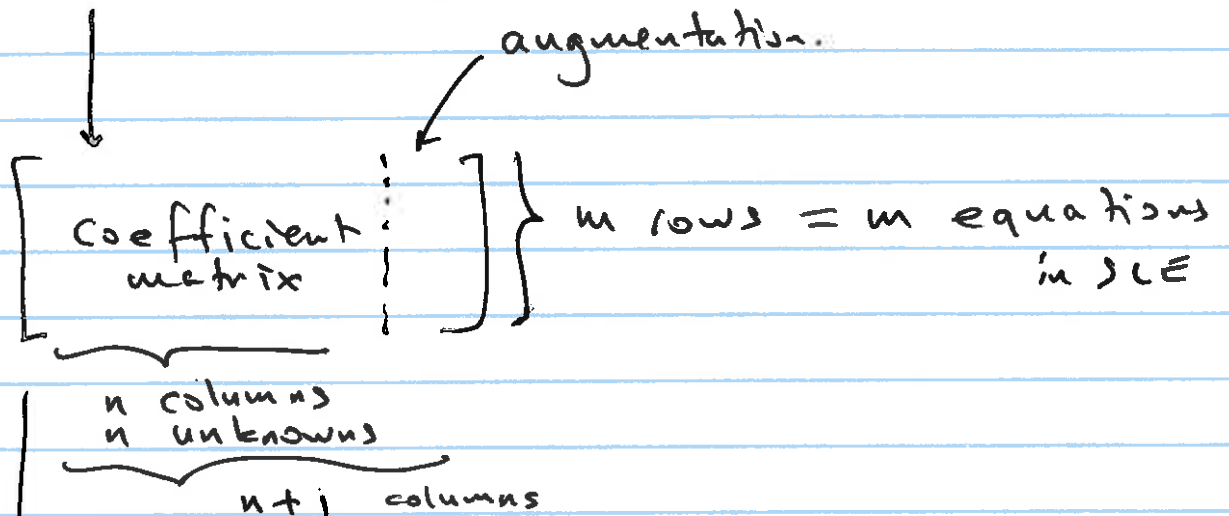
- Each of All zero rows must appear below all rows with a nonzero entry
- Leading entry of a row is to the left of the leading entry of a row below
- For each leading entry of a row the entries below are 0
- Each column with a leading 1 has 0's in the rest of the column
- Each leading entry is 1.

$$\begin{bmatrix} 0 & 1^* & 5 & 0 & 3 & 0 & -1 \\ 0 & 0 & 0 & 1^* & 4 & 0 & 6 \\ 0 & 0 & 0 & 0 & 0 & 1^* & 7 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

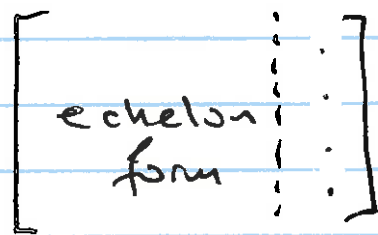
Thus For every matrix there exists a unique RREF.

# How do we read solutions from RREF

SLE



Row Reduce

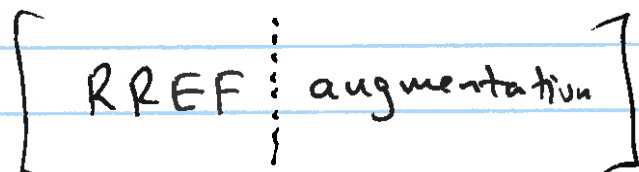


Here decide whether the system is consistent or inconsistent

if consistent row reduce further

$[0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ ; \ a \neq 0]$   
need at least one such row for inconsistency

$[0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ ; \ 0]$   
says nothing

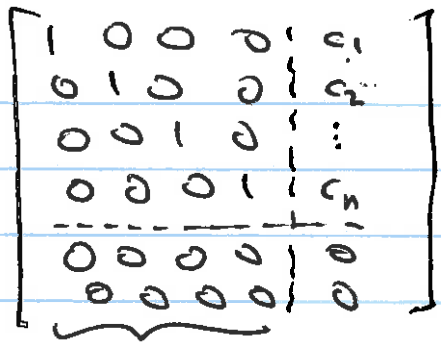


PTO

Consistent

Case 1

Each column of RREF of the coeff. matrix has a leading 1 (not augmentation)



n columns  $\leftrightarrow$  n unknowns:  $x_1, x_2, \dots, x_n$

Solution is unique

$$\begin{aligned} x_1 &= c_1 \\ x_2 &= c_2 \\ &\vdots \\ x_n &= c_n \end{aligned}$$

Consistent

Case 2

There are some columns of RREF of the main coeff. matrix (not including the augmentation part) without a leading 1 of any row.



pivot columns (have leading 1's)

no leading 1's These unknowns must be assigned as free variables.

Then solve all of the unknowns in terms of free variables x number

EXS

a

$$\left[ \begin{array}{ccc|c} 1 & 5 & 0 & 5 \\ 0 & 0 & 1 & 6 \\ 0 & 0 & 0 & 7 \end{array} \right]$$

Inconsistent.  
no solution.

b

$$\begin{array}{ccc} x_1 & x_2 & x_3 \\ \left[ \begin{array}{ccc|c} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 6 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right] \end{array}$$

$$\rightarrow \left. \begin{array}{l} x_1 = 5 \\ x_2 = -2 \\ x_3 = 6 \end{array} \right\} \text{unique sol}^n.$$

c

$$\left[ \begin{array}{ccccc|c} x_1 & x_2 & x_3 & x_4 & x_5 & \\ \hline 1 & 0 & -5 & 0 & 0 & 3 \\ 0 & 1 & 4 & -1 & 0 & 6 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

↑ ↑  
Free variables

$$x_1 - 5x_3 = 3$$

$$x_1 = 3 + 5x_3$$

$$x_2 + 4x_3 - x_4 = 6$$

$$x_2 = 6 - 4x_3 + x_4$$

$$x_1 = 3 + 5x_3$$

$$x_2 = 6 - 4x_3 + x_4$$

$$x_3 = \text{free to choose}$$

$$x_4 = \text{free to choose}$$

$$x_5 = 0$$

Infinitely many sol<sup>n</sup>. / parametric sol<sup>n</sup>.

$$\begin{aligned}
 E_x \quad x + 2y + 3z &= 0 \\
 2x - y - z &= 1 \\
 5x \quad \quad + z &= 2
 \end{aligned}$$

$$\left[ \begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 2 & -1 & -1 & 1 \\ 5 & 0 & 1 & 2 \end{array} \right] \xrightarrow{R_2 - 2R_1} \left[ \begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & -5 & -7 & 1 \\ 5 & 0 & 1 & 2 \end{array} \right]$$

$$\begin{array}{l}
 \downarrow R_2 \\
 \xrightarrow{R_3 - 5R_1} \\
 \downarrow R_3
 \end{array}
 \left[ \begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & -5 & -7 & 1 \\ 0 & -10 & -14 & 2 \end{array} \right] \xrightarrow{R_3 - 2R_2} \left[ \begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & -5 & -7 & 1 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

$$\begin{array}{l}
 \xrightarrow{-\frac{1}{5}R_2} \\
 \downarrow R_1 \\
 \downarrow R_3
 \end{array}
 \left[ \begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & 1 & \frac{7}{5} & -\frac{1}{5} \\ 0 & 0 & 0 & 0 \end{array} \right] \xrightarrow{R_1 - 2R_2} \left[ \begin{array}{ccc|c} 1 & 0 & \frac{1}{5} & \frac{2}{5} \\ 0 & 1 & \frac{7}{5} & -\frac{1}{5} \\ 0 & 0 & 0 & 0 \end{array} \right]$$

$\uparrow$  pivot       $\uparrow$  RREF       $\uparrow$  Free z

$$\begin{aligned}
 x + \frac{1}{5}z &= \frac{2}{5} \\
 x &= \frac{2}{5} - \frac{1}{5}z
 \end{aligned}$$

$$\begin{aligned}
 y + \frac{7}{5}z &= -\frac{1}{5} \\
 y &= -\frac{1}{5} - \frac{7}{5}z
 \end{aligned}$$

$$\begin{aligned}
 x &= \frac{2}{5} - \frac{1}{5}z \\
 y &= -\frac{1}{5} - \frac{7}{5}z \\
 z &= \text{free to choose}
 \end{aligned}$$

1.3

Def

$$\mathbb{R}^n = \left\{ \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_n \end{bmatrix} : c_1, c_2, \dots, c_n \in \mathbb{R} \right\}$$

$$\begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_n \end{bmatrix} + \begin{bmatrix} d_1 \\ d_2 \\ \vdots \\ d_n \end{bmatrix} = \begin{bmatrix} c_1 + d_1 \\ c_2 + d_2 \\ \vdots \\ c_n + d_n \end{bmatrix}$$

$$k \cdot \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_n \end{bmatrix} = \begin{bmatrix} kc_1 \\ kc_2 \\ \vdots \\ kc_n \end{bmatrix}$$

$\mathbb{R}^n$ , with two operations  $+$ ,  $\cdot$ , is going to be called a vector space.

$$\begin{bmatrix} 2 \\ 3 \end{bmatrix} + \begin{bmatrix} -1 \\ 4 \end{bmatrix} = \begin{bmatrix} 1 \\ 7 \end{bmatrix}$$

$$\begin{bmatrix} 2 \\ 5 \\ 0 \end{bmatrix} \overset{\uparrow}{\mathbb{R}^3} + \begin{bmatrix} 3 \\ 7 \end{bmatrix} \overset{\uparrow}{\mathbb{R}^2} = \underline{\text{Can't add them.}}$$

six
Compare handwriting
letter b

$$5 \begin{bmatrix} 6 \\ -1 \\ a \end{bmatrix} - 3 \begin{bmatrix} 6 \\ 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 30 - 3b \\ -14 \\ 5a - 12 \end{bmatrix}$$

Compare handwriting of similar symbols

$$2z + 6b + cC + xX - uU + 5sS$$