1/28/2020

①

Matrix RREM

1.2 Row Reduced Echelon Form RREF :

Echelon

- Each of All zero rows must appear below all rows with a nonzero entry
- Leading entry of a row is to the left of the leading entry of a row below
- For each leading entry of a row the entries below are 0
- Each column with a leading 1 has 0's in the rest of the column
- Each leading entry is 1.

$$\begin{bmatrix} 0 & 1^* & 5 & 0 & 3 & 0 & -1 \\ 0 & 0 & 0 & 1^* & 4 & 0 & 6 \\ 0 & 0 & 0 & 0 & 0 & 1^* & 7 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Thus For every matrix there exists a unique RREF.

How do we read solutions from RREF

SLE

↓

augmentation

$\left[\begin{array}{c|c} \text{coefficient matrix} & \end{array}\right]$ } m rows = m equations in SLE

n columns
n unknowns

n+1 columns

Row Reduce

↓

$\left[\begin{array}{c|c} \text{echelon form} & \vdots \end{array}\right]$

Here decide whether the system is consistent or inconsistent

$[0\ 0\ 0\ 0\ 0\ 0 \mid a \neq 0]$
need at least one such row for inconsistency

$\underbrace{[0\ 0\ 0\ 0\ 0\ 0\ 0 \mid 0]}_{\text{says nothing}}$

if consistent row reduce further

↓

$\left[\begin{array}{c|c} \text{RREF} & \text{augmentation} \end{array}\right]$

PTO

Consistent **Case 1** Each column of RREF of the coeff. matrix has a leading 1 (not augmentation)

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & c_1 \\ 0 & 1 & 0 & 0 & c_2 \\ 0 & 0 & 1 & 0 & \vdots \\ 0 & 0 & 0 & 1 & c_n \\ \hline 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

n columns $\Leftrightarrow$ n unknown : $x_1, x_2 \cdots x_n$

Solution is unique
$$x_1 = c_1$$
$$x_2 = c_2$$
$$\vdots$$
$$x_n = c_n$$

Consistent **Case 2** There are some columns of RREF of the main coeff. matrix (not including the augmentation part) without a leading 1 of any row.

$$\left[\begin{array}{cccccc|} 1 & * & 0 & 0 & 0 & * \\ 0 & 0 & 1 & * & 0 & * \\ 0 & 0 & 0 & 0 & 1 & * \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

pivot columns (have leading 1's)

no leading 1's These unknowns must be assigned as free variables.

Then solve all of the unknowns in terms of free variables + number

Exs

a

$$\left[\begin{array}{ccc|c} 1 & 5 & 0 & 5 \\ 0 & 0 & 1 & 6 \\ \boxed{0} & \boxed{0} & \boxed{0} & \boxed{7} \end{array}\right] \longrightarrow$$

Inconsistent. no solution.

b

$$\begin{array}{ccc} x_1 & x_2 & x_3 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 6 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \longrightarrow \left.\begin{array}{l} x_1 = 5 \\ x_2 = -2 \\ x_3 = 6 \end{array}\right\} \text{unique sol}^n.$$

c

$$\begin{array}{ccccc} x_1 & x_2 & x_3 & x_4 & x_5 \end{array}$$

$$\left[\begin{array}{ccccc|c} \boxed{1} & 0 & -5 & 0 & 0 & 3 \\ 0 & \boxed{1} & 4 & -1 & 0 & 6 \\ 0 & 0 & 0 & 0 & \boxed{1} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

Free variables

$$x_1 - 5x_3 = 3$$
$$x_1 = 3 + 5x_3$$

$$x_2 + 4x_3 - x_4 = 6$$
$$x_2 = 6 - 4x_3 + x_4$$

$x_1 = 3 + 5x_3$
$x_2 = 6 - 4x_3 + x_4$
x_3 = free to choose
x_4 = free to choose
$x_5 = 0$

Infinitely many sol^{ns}. / parametric sol^n.

Ex

$$\begin{aligned} x + 2y + 3z &= 0 \\ 2x - y - z &= 1 \\ 5x \quad\quad + z &= 2 \end{aligned}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 2 & -1 & -1 & 1 \\ 5 & 0 & 1 & 2 \end{array}\right] \xrightarrow[R_2 - 2R_1 \to R_2]{} \left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & -5 & -7 & 1 \\ 5 & 0 & 1 & 2 \end{array}\right]$$

$$\xrightarrow[R_3 - 5R_1 \to R_3]{} \left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & -5 & -7 & 1 \\ 0 & -10 & -14 & 2 \end{array}\right] \xrightarrow[R_3 - 2R_2 \to R_3]{} \left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & -5 & -7 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$\xrightarrow[-\frac{1}{5}R_2]{} \left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & 1 & \frac{7}{5} & -\frac{1}{5} \\ 0 & 0 & 0 & 0 \end{array}\right] \xrightarrow[R_1 - 2R_2 \to R_1]{} \left[\begin{array}{ccc|c} 1 & 0 & 1/5 & \frac{2}{5} \\ 0 & 1 & 7/5 & -1/5 \\ \underbrace{0 \quad 0 \quad 0}_{\text{RREF}} & & & 0 \end{array}\right]$$

pivot

Free z

$$x + \frac{1}{5}z = \frac{2}{5}$$

$$x = \frac{2}{5} - \frac{1}{5}z$$

$$y + \frac{7}{5}z = -\frac{1}{5}$$

$$y = -\frac{1}{5} - \frac{7}{5}z$$

$$\begin{aligned} x &= \frac{2}{5} - \frac{1}{5}z \\ y &= -\frac{1}{5} - \frac{7}{5}z. \\ z &= \text{free to choose} \end{aligned}$$

1.3

Defn

$$\mathbb{R}^n = \left\{ \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_n \end{bmatrix} : c_1, c_2 \ldots c_n \in \mathbb{R} \right\}$$

$$\begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_n \end{bmatrix} + \begin{bmatrix} d_1 \\ d_2 \\ \vdots \\ d_n \end{bmatrix} = \begin{bmatrix} c_1 + d_1 \\ c_2 + d_2 \\ \vdots \\ c_n + d_n \end{bmatrix}$$

$$k \cdot \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_n \end{bmatrix} = \begin{bmatrix} kc_1 \\ kc_2 \\ \vdots \\ kc_n \end{bmatrix}.$$

$\mathbb{R}^n$, with the two operations $+$, $\cdot$, is going to be called a vector space.

$$\begin{bmatrix} 2 \\ 3 \end{bmatrix} + \begin{bmatrix} -1 \\ 4 \end{bmatrix} = \begin{bmatrix} 1 \\ 7 \end{bmatrix}$$

$$\underset{\uparrow \atop \mathbb{R}^3}{\begin{bmatrix} 2 \\ 5 \\ 0 \end{bmatrix}} + \underset{\uparrow \atop \mathbb{R}^2}{\begin{bmatrix} 3 \\ 7 \end{bmatrix}} = \text{Can't add them.}$$

six

compare handwriting

letter b

$$5\begin{bmatrix} 6 \\ -1 \\ a \end{bmatrix} - 3\begin{bmatrix} b \\ 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 30 - 3b \\ -14 \\ 5a - 12 \end{bmatrix}$$

Compare handwriting of similar symbols

$$2z + 6b + cG + x\,X - uv + 5sS$$