1.1 + 1.2

$$\text{SLE} \longrightarrow \left[\begin{array}{c|c} \text{coefficient matrix} & \text{Augmentation} \end{array}\right]$$

* **Elementary row operations**

- Add a multiple of a row to another row
- Interchange 2 rows
- multiply a row with a $\neq 0$ real #.

Defn 2 SLE's are called row-equivalent if there is a finite sequence of elementary row operations which takes one to the other.

Thm: If two SLE's are row equivalent then they have the same solution set.

Ex Solve

$$x + y = 5$$
$$x - y = 3$$
$$2x + y = 3$$

$$\left[\begin{array}{cc|c} 1 & 1 & 5 \\ 1 & -1 & 3 \\ 2 & 1 & 3 \end{array}\right] \xrightarrow[-R_1+R_2 \to R_2]{} \left[\begin{array}{cc|c} 1 & 1 & 5 \\ 0 & -2 & -2 \\ 2 & 1 & 3 \end{array}\right] \xrightarrow[R_3 - 2R_1 \to R_3]{} \left[\begin{array}{cc|c} 1 & 1 & 5 \\ 0 & -2 & -2 \\ 0 & -1 & -7 \end{array}\right]$$

$$-\tfrac{1}{2} R_2 \longrightarrow \left[\begin{array}{cc|c} 1 & 1 & 5 \\ 0 & 1 & 1 \\ 0 & -1 & -7 \end{array}\right] \xrightarrow[R_3 + R_2 \to R_3]{} \left[\begin{array}{cc|c} 1 & 1 & 5 \\ 0 & 1 & 1 \\ 0 & 0 & -6 \end{array}\right] \longrightarrow$$

$0 \cdot x + 0 \cdot y = -6$

never true

$\boxed{\text{INCONSISTENT}}$

Ex Solve

$$\begin{aligned} x+y+z&=5\\ x-y+z&=3\\ x+z&=4 \end{aligned}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 5\\ 1 & -1 & 1 & 3\\ 1 & 0 & 1 & 4 \end{array}\right] \xrightarrow[R_2 - R_1 \to R_2]{} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 5\\ 0 & -2 & 0 & -2\\ 1 & 0 & 1 & 4 \end{array}\right]$$

$$\xrightarrow[R_3 - R_1 \to R_3]{} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 5\\ 0 & -2 & 0 & -2\\ 0 & -1 & 0 & -1 \end{array}\right] \xrightarrow[\substack{-\frac{1}{2}R_2\\ -R_3}]{} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 5\\ 0 & 1 & 0 & 1\\ 0 & 1 & 0 & 1 \end{array}\right]$$

$$\xrightarrow[R_3 - R_2 \to R_3]{} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 5\\ 0 & 1 & 0 & 1\\ 0 & 0 & 0 & 0 \end{array}\right] \xrightarrow[R_1 - R_2 \to R_1]{} \left[\begin{array}{ccc|c} 1^* & 0 & 1 & 4\\ 0 & 1^* & 0 & 1\\ 0 & 0 & 0 & 0 \end{array}\right]$$

pivots

leading 1's

Free variable

$$\begin{aligned} x+z&=4\\ y&=1 \end{aligned} \quad \longrightarrow \quad \begin{aligned} x&=4-z\\ y&=1\\ z&=\text{free to choose (parameter)} \end{aligned}$$

∞ly many solutions. For each choice of z, I get a different answer/solution.

$\underbrace{(4,1,0),\ (3,1,1)}_{\text{some solutions}}$, $\underbrace{(4-z,\ 1,\ z),\ z\in\mathbb{R}}_{\text{all solutions.}}$

1.1

Exc #20, 22 Find values of h which makes system consistent

#20

$$\left[\begin{array}{cc|c} 1 & h & -3 \\ -2 & 4 & 6 \end{array}\right] \xrightarrow[2R_1+R_2 \to R_2]{} \left[\begin{array}{cc|c} 1 & h & -3 \\ 0 & 4+2h & 0 \end{array}\right] \dashrightarrow \quad 0\cdot x + (4+2h)y = 0$$

if $h=-2$ $\left[\begin{array}{cc|c} 1 & -2 & -3 \\ 0 & 0 & 0 \end{array}\right]$ consistent (infinitely many solutions.)

$h \neq -2$ $\left[\begin{array}{cc|c} 1 & h & -3 \\ 0 & \underset{\neq 0}{4+2h} & 0 \end{array}\right]$ Consistent (unique solution)

For all values of h, this system is consistent

#22

$$\left[\begin{array}{cc|c} 2 & -3 & h \\ -6 & 9 & 5 \end{array}\right] \xrightarrow[3R_1+R_2 \to R_2]{} \left[\begin{array}{cc|c} 2 & -3 & h \\ \underbrace{0 \quad\quad 0 \quad\quad}_{*} & & 3h+5 \end{array}\right]$$

If $3h+5=0$ i.e., $h=-\frac{5}{3}$ then system is consistent $\left[\begin{array}{cc|c} 2 & -3 & -\frac{5}{3} \\ 0 & 0 & 0 \end{array}\right]$

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if $3h+5 \neq 0$ $h \neq -\frac{5}{3}$ then system is inconsistent

$*$ $0 = 0x + 0\cdot y = 3h+5 \neq 0$

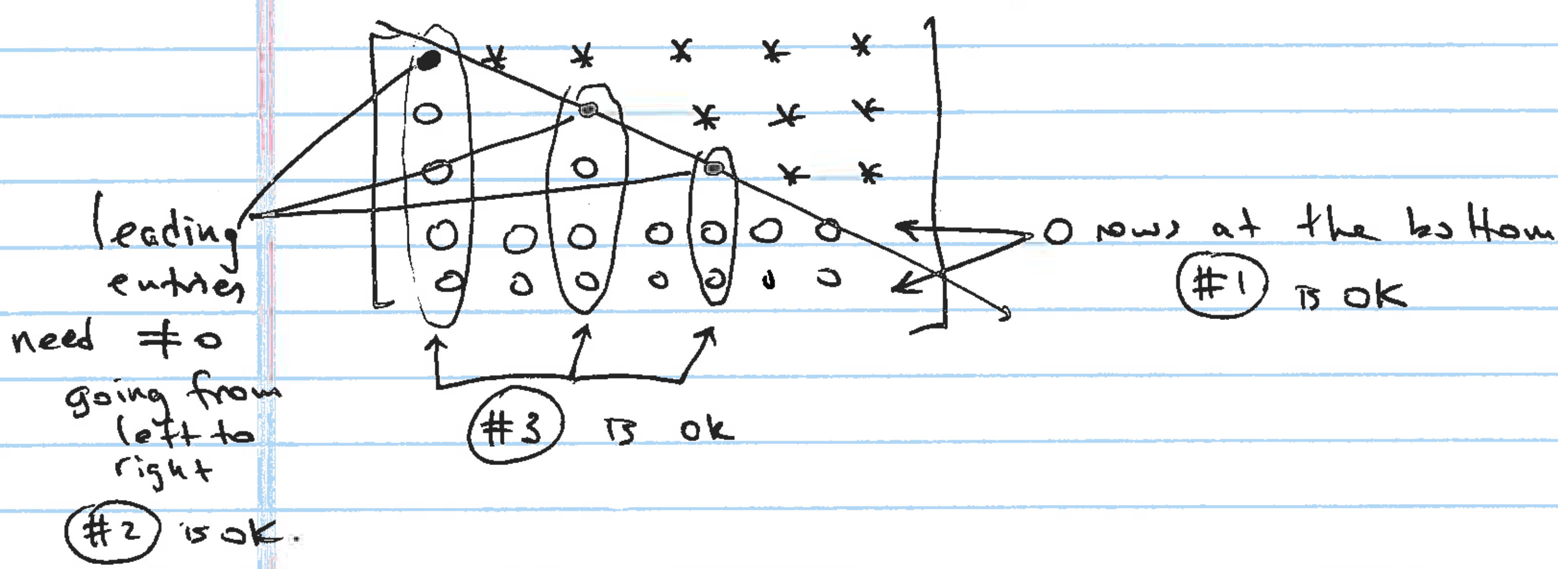
1.2 Leading entry of a row :
= the leftmost non-zero entry

Echelon form : 3 conditions

#1. All zero rows must appear below all rows with a non-zero entry.

#2. Leading entry of a row is to the left of the leading entry of a row below

#3. For each leading entry of a row the entries below (in the same column) are 0.



Ex — In Echelon form? Why?

a) $\begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 3 \\ 0 & 0 & 1 \end{bmatrix}$ No — leading 1's: # 2 fails

b) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ No — #1 fails, zero row is not at the bottom.

leading entry of row 1

c) $\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 1 & 2 & 0 \\ 1 & 0 & 3 & 0 \end{bmatrix}$ No — #3 fails

d) $\begin{bmatrix} 3 & 5 & 2 & 1 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ Yes — 1✓, 2✓, 3✓

leading entries

0 row at the bottom. #1✓

3 2 3

#2✓

$\begin{matrix} 3 & 5 & 2 & 1 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 \end{matrix}$

#3 is true as well.