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CHAPTER I

Which ones are linear eq^{n}'s?

1.1

$xy^2 = 5$ no

$\sqrt{x} + y = 3$ no

$x - 3y = 6$ yes

$e^x - y = 7$ no

$x = 1$ yes

$x^2 + x - 1 = 0$ no

$\cos x = 0$ no

Def^n. An equation of the form
$a_1x_1 + a_2x_2 + \cdots + a_nx_n = b$
(where $x_1, x_2 \ldots x_n$ are unknowns
& $a_1, a_2 \ldots a_n$ are real #'s)
is called a linear equation.

- SLE System of Linear equations is a collection of one or more linear equations.

Ex. ①

$$\left.\begin{aligned} x + 3y &= 5 \\ x - y &= 7 \end{aligned}\right\} \text{ SLE}$$

②

$$\left.\begin{aligned} x + y - 3z &= 6 \\ x + z &= 7 \\ y - z &= 5 \end{aligned}\right\} \text{ SLE}$$

Def A solution of an SLE in n unknowns $x_1, x_2 \ldots x_n$ is

$$\begin{cases} x_1 = c_1 \\ x_2 = c_2 \\ x_3 = c_3 \\ \vdots \\ x_n = c_n \end{cases} \quad \text{OR} \quad (x_1, x_2 \ldots x_n) = (c_1, c_2 \ldots c_n)$$

unknowns numbers

* which satisfies the given SLE

Ex

$$\left.\begin{aligned} x - y &= 4 \\ 2x + y &= 5 \end{aligned}\right\} \text{SLE} \qquad \underline{\text{Sol'n}} \quad \left.\begin{aligned} x &= 3 \\ y &= -1 \end{aligned}\right\}$$

OR

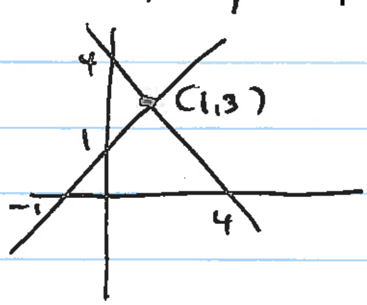
$$(x, y) = (3, -1)$$

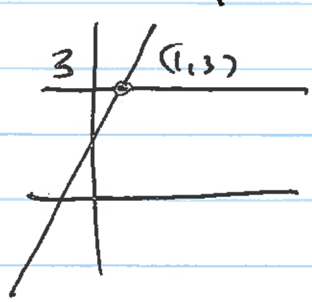
Defn · Given an SLE, the set of all solutions is called the solution set.

· Two SLE's are called equivalent if they have identical solution sets.

Ex

$$\begin{aligned} x + y &= 4 \\ -2x + y &= 1 \end{aligned} \quad \xleftrightarrow{\text{equivalent}} \quad \begin{aligned} x - y &= -2 \\ y &= 3 \end{aligned}$$

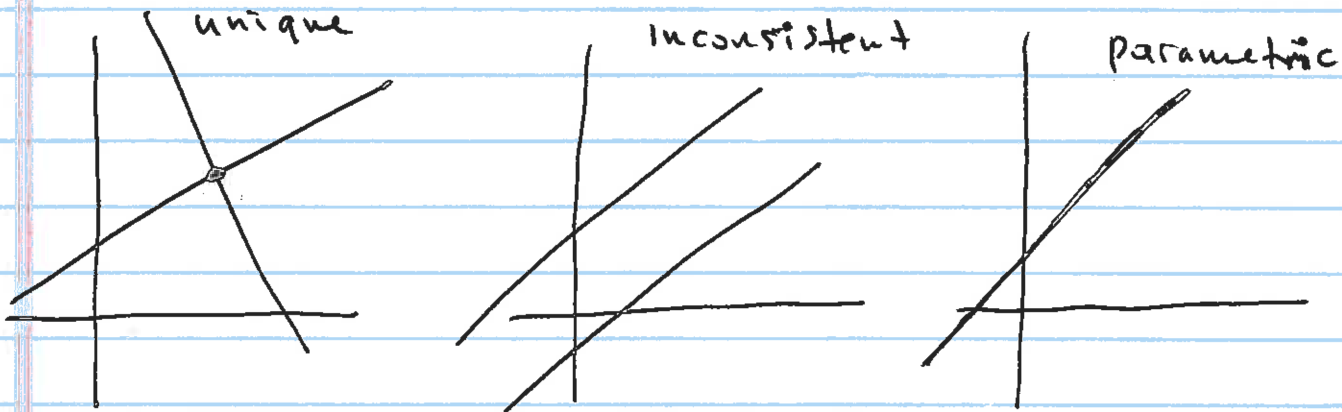




In General for 2-unknowns

$$ax + by = A \qquad \underbrace{a, b, c, d}_{\text{not all } 0},\ A,\ B \in \mathbb{R}$$
$$cx + dy = B$$

(element of)



intersecting at 1 pt (transversally) — parallel — identical

Theorem: Given an SLE (non-empty), its solution set has

Inconsistent → a) No solution OR

Consistent { b) Exactly one solution OR
c) Infinitely many solutions.

unique sol^n

parametric solution

MATRICES of SLE's

Ex

$$\left.\begin{aligned} 2x+3y-z &= 4 \\ x-y \quad &= 3 \\ y+4z &= 0 \end{aligned}\right\}$$

Textbook doesn't show the dotted line.

$$\left[\begin{array}{ccc:c} 2 & 3 & -1 & 4 \\ 1 & -1 & 0 & 3 \\ 0 & 1 & 4 & 0 \end{array}\right]$$

Coefficient matrix

Augmentation

Talked about this in class

How do we solve SLEs:

- Multiply an equation with a nonzero #
- Add a multiple of an equation to another
- Interchange equations.

Correspond to

For matrices

- Multiply a row (all of it) with a non-zero #
- Add a multiple of a row to another row
- Interchange rows.

So: We will work with matrices!
Shorter / write less / keep track / Procedural: computer can do i

This example is to show the flow of the procedure

Ex Solve SLE
$$\begin{cases} x+y-z=-2 \\ x-y+z=0 \\ 2x+y+3z=9. \end{cases}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & -2 \\ 1 & -1 & 1 & 0 \\ 2 & 1 & 3 & 9 \end{array}\right] \xrightarrow[-R_1+R_2 \to R_2]{} \left[\begin{array}{ccc|c} 1 & 1 & -1 & -2 \\ 0 & -2 & 2 & 2 \\ 2 & 1 & 3 & 9 \end{array}\right]$$

$$\xrightarrow[R_3-2R_1 \to R_3]{} \left[\begin{array}{ccc|c} 1 & 1 & -1 & -2 \\ 0 & -2 & 2 & 2 \\ 0 & -1 & 5 & 13 \end{array}\right] \xrightarrow[-\frac{1}{2}R_2]{} \left[\begin{array}{ccc|c} 1 & 1 & -1 & -2 \\ 0 & 1 & -1 & -1 \\ 0 & -1 & 5 & 13 \end{array}\right]$$

$$\xrightarrow[R_1-R_2 \to R_1]{} \left[\begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 0 & 1 & -1 & -1 \\ 0 & -1 & 5 & 13 \end{array}\right] \xrightarrow[R_2+R_3 \to R_3]{} \left[\begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 4 & 12 \end{array}\right]$$

$$\xrightarrow[\frac{1}{4}R_3]{} \left[\begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 1 & 3 \end{array}\right] \xrightarrow[R_3+R_2 \to R_2]{} \left[\begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

unique sol^{n}.
$$\begin{cases} x = -1 \\ y = 2 \\ z = 3 \end{cases}$$