MATH 2850 **Practice questions for Midterm 2**.

Your test is 50 minutes long. All of these 17 questions should be doable in 3 hours, so in average each should take 10 minutes. (Caution some are 5-minute and some are 15-minute problems.)

Problem 1. Let $f(x,y) = x^3 - 3x^2 + y^3 - 27y : \mathbf{R}^2 \to \mathbf{R}$.

a. Find all critical points of f.

b. Find all of the local maxima, local minima, and saddle points of f.

Problem 2. **a**. Evaluate $\int_0^2 \int_0^{2x} \int_0^{3y} xyz\, dz\, dy\, dx$.

b. Sketch the domain of integration of the integral above.

Problem 3. Consider the integral $\int_0^3 \int_{x^2}^9 4xe^{(y^2)}\, dy\, dx$.

a. Sketch the domain of integration.

b. Reverse the order of integration.

c. Find the (numerical) value of the integral above.

Problem 4. Find the maximum and minimum values of $f(x,y) = x^2 + xy + y^2$ on the domain defined by $g(x,y) = \frac{1}{2}x^2 + \frac{1}{2}y^2 \leq 1$. Indicate any theorem you use in your solution, where and how it is used.

Problem 5. Let $\mathbf{X}(t) = (4\cos t, 4\sin t, 3t)$ for $0 \leq t \leq 2\pi$.

a. For the parametric motion defined by $\mathbf{X}(t)$, find its velocity, acceleration and speed.

b. Find a parametric equation for the tangent line to $\mathbf{X}(t)$ when $t = \pi$.

c. Find the length of $\mathbf{X}(t)$.

d. Sketch the curve $\mathbf{X}(t)$, and include the coordinates of its end points.

Problem 6. Find the first order Taylor polynomial for $f(x,y) = x^2\cos 3y + xy^2$ near $(a,b) = (3,0)$.

b.Use part (a) to approximate $f(3.04, 0.03)$.

c. Find the second order Taylor polynomial for f near $(a,b) = (3,0)$.

Problem 7 a. Sketch the domain of integration of the integral and reverse the order of integration: $\int_0^1 \int_0^{2x} e^x \cos y\, dy\, dx$.

b. Sketch the domain of integration of the integral $\int_0^{\pi/2} \int_y^{4-y} x\sin y\, dx\, dy$ and calculate the value of the integral.

Problem 8. Let $h(x,y) = x^2 - y^2 - 2y$.

a. Find all critical points of $h(x,y)$ on $\mathbf{R}^2$. Determine whether each critical point is a relative maximum, minimum, or saddle point.

b. Find the points at which the function $h(x,y) = x^2 - y^2 - 2y$ attains its maximum and minimum values on the circle $x^2 + y^2 = 1$. State the maximum and minimum values.

Problem 9.

Let $f(x,y,z) = x^2y^3 - yz + 2xz$

a. Calculate ∇f.

b. Calculate the directional derivative of f at $(x,y,z) = (1,0,2)$ in the direction of the vector $(2,-2,1)$.

c. In which direction does f decrease fastest at $(1,0,2)$?

d. Find an equation for tangent plane to the level set $x^2y^3 - yz + 2xz = 4$ at $(1,0,2)$.

Problem 10. Find all critical points of $f(x,y) = x^3 - 2xy + y^2 - x : \mathbf{R}^2 \to \mathbf{R}$, and determine whether each critical point is a local maximum, a local minimum or a saddle point by using the Second Derivative Test.

Problem11. **a**. Why do the equations $\begin{cases} x^2 + y^2 + z^2 - w = 1 \\ x - y + z + w^2 = 5 \end{cases}$ implicitly define x and y as functions of z and w near the point $(x,y,z,w) = (1,1,1,2) = \mathbf{p}$?

b. If $g(z,w) = (x,y)$ is the solution function near $\mathbf{p}$, compute the derivative matrix g' near $\mathbf{p}$ in terms of x, y, z and w.

Problem 12. For each of the following iterated integrals, sketch the domain of integration of the integral and find the value of the integral.

a. $\int_0^2 dx \int_{2x}^4 dy \int_0^y e^z dz$

b. $\int_0^2 dx \int_{2x}^4 \cos(y^2) dy$

Problem 13. Let B be the subset of $\mathbf{R}^3$, defined by
$y^2 \geq z$,
$x + y \leq 1$,
$x \geq 0$,
$y \geq 0$, and
$z \geq 0$.
a. Sketch the region B.
b. Calculate the volume of B.
c. Calculate $\iiint_B 12yz\, dV$

Problem14. Sketch the region D defined by $x^2 + y^2 \leq 9$, $x \geq 0$ and $y \geq 0$ in $\mathbf{R}^2$, describe the region D by using polar coordinates, and calculate $\iint_D \sqrt{x^2 + y^2}\ dx\, dy$.

Problem 15. Let a transformation T from the uv-plane to the xy-plane be defined by $x = u + v$ and $y = u - v$. Let R_{uv} be the rectangular region given by $0 \leq u \leq 2$, and $0 \leq v \leq 1$ in the uv-plane.
a. Find and sketch the region $R_{xy} = T(R_{uv})$, the image of R_{uv} under the transformation T.
b. Find $\frac{\partial(x,y)}{\partial(u,v)}$.
c. Transform $\int_{R_{xy}} xy\, dx\, dy$ to an integral over R_{uv}. Write the integral as an iterated integral, but do not evaluate it.

Problem 16. For the curve L parametrized by $g(t) = (2t, \ln t, t^2)$, for $1 \leq t \leq e^2$, calculate the following.
a. the velocity
b. the speed
c. The length of L
d. The equation of the tangent line to L when $t = e$.

Problem 17. Let $S = \{(x,y) : 2y = x^2\}$ and $(0,a)$ be a given point on the $y - axis$ in $\mathbf{R}^2$. Find the closest point(s) of S to $(0,a)$ in terms of a.

HINTS: It is easier to minimize the square of the distance function from $(0,a)$. Also, you will have different types of answers depending on whether a is large or small.