MATH 2850

Some Practice questions for the Final Exam

Problem 1. Sketch the domain of integration of the following integral and express it in spherical coordinates as an iterated integral with bounds. Compute the value of this integral.

$$\int_0^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} z(x^2+y^2)\,dzdydx$$

Problem 2. **a**. Give a parametrization of the line segment C from the $(1,0,0)$ to $(1,3,4)$.

b. Calculate the line integral $\int_C \mathbf{F}\cdot\mathbf{ds}$ along the curve C of part (**a**), where $\mathbf{F}(x,y,z) = (-y,-z,x)$.

Problem 3. **a**. Give a parametrization of the cylinder S described by $x^2+y^2 = 4$ and $0 \le z \le 3$. Sketch the surface and specify the parametrization domain.

b. Find a closed equation $(Ax+By+Cz = D)$ for the tangent plane to S at the point $(\sqrt{3},1,2)$.

Problem 4.**a**. Calculate the divergence of $\mathbf{F}(x,y,z) = xyz\mathbf{i} + x\cos y\mathbf{j} + e^{2z}\mathbf{k}$.

b. Prove that $\nabla\times\nabla f = 0$, for every twice continuously differentiable function $f : \mathbf{R}^3 \to \mathbf{R}$.

Problem 5. Let B be the subset of $\mathbf{R}^3$, defined by $x+y \le 2,\ z \le 4-x^2$ and $x \ge 0,\ y \ge 0,\ z \ge 0$.

a. Sketch the region B. **b**. Calculate the volume of B.

Problem 6. **a**. Is $\mathbf{F}(x,y,z) = (z+yz, xz, x+xy)$ a conservative vector field? If it is, find a potential function for $\mathbf{F}$. If it is not, explain why.

b. Calculate the line integral $\int_C \mathbf{F}\cdot d\mathbf{s}$, where C is the curve given by $g(t) = (\cos^3 t, \sin^4 t, \ln(1+t)) \ : [0,\pi] \to \mathbf{R}^3$.

Problem 7. Let $g(u,v) = (u,v,v^2-u^2)$ parametrize a surface S for $0 \le u \le 2$ and $0 \le v \le 2$.

a. Calculate $\int_S \sqrt{1+4z+8x^2}\ dS$.

b. Calculate the *flux* through the surface S, $\iint_S \mathbf{F}\cdot d\mathbf{S}$, where $\mathbf{F}(x,y,z) = (2x,z,y)$.

Problem 8. Verify that the Stokes' Theorem holds (by computing both integrals and showing that they are equal) for the vector field $\mathbf{F}(x,y,z) = (x,y,0)$ and for the surface S parametrized by $g(u,v) = (u,v,u^2+v^2)$ for $u^2+v^2 \le 4$. Sketch S and its border.

Problem 9. Find $\int_{C_i} (x^2-y^3)dx + (x^3-y^2)dy$, for each of the following curves.

a. C_1 is the unit circle $x^2+y^2 = 1$, traced counterclockwise.

b. C_2 is the part of the unit circle $x^2+y^2 = 1$ in the upper half-plane $(y \ge 0)$ traced clockwise from $(-1,0)$ to $(1,0)$.

Problem 10. Calculate $\iint_D (2x+y)\cos(y-x)dA$ where D is the parallelogram with vertices $(0,0),(2,2),(3,0)$ and $(1,-2)$. (Hint: Find the equations of the edges, and make a 2 variable substitution.)

Problem 11. Verify Gauss' Theorem for the 3-dimensional region $D = \{(x,y,z) : x^2+y^2 \le 1 \le z \le 5\}$ and vector field $\mathbf{F}(x,y,z) = x^2\mathbf{i} + y\mathbf{j} + z\mathbf{k}$.

Problem 12. Let $\mathbf{G} = (2xy - e^x, x^2 + 4y)$.

a. Is $\mathbf{G}$ a conservative vector field? Explain why, if it is not. Find a potential function for it, if it is conservative.

b. Calculate $\int_C \mathbf{G} \cdot \mathbf{ds}$ where C is the curve from $(0,0)$ along x-axis to the point $(3,0)$ and then following the circle $x^2+y^2 = 9$ counterclockwise to the point $(0,-3)$.

Problem 13. Compute $\oint_C x^2dx + x^2ydy,$ where the curve C traces the boundary of the triangle with vertices $(0,0),$ $(2,2)$ and $(0,2),$ counterclockwise.

Problem 14. Convert the following integral to polar coordinates, and compute it.

$$\int_0^2 \int_0^{\sqrt{4-x^2}} (2x^2+y^2)dydx$$

Problem 15.

a. Give a parametrization for the surface $z^2 = x^2+y^2$ where $0 \le z \le 3$.

b. Set up the area integral $\iint_S 1dS$ as an iterated integral with bounds, according to the parametrization you obtained, and evaluate this integral.

Problem 16. Compute the flux integral $\iint_S \nabla \times \mathbf{F} \cdot d\mathbf{S}$, where $\mathbf{F}(x,y,z) = (x^2, z^2, y^2),$ and S is the hemisphere $z = \sqrt{1-x^2-y^2}$. Assume that S is oriented with normal pointing away from the origin.

Problem 17. For the curve L given by $g(t) = (t, t^2, \frac{2}{3}t^3),\ 0 \le t \le 1,$ calculate the following.

a. $\int_L (x^2+y)\, ds$

b. $\int_L e^x dx + zdy + \sin z dz$

Problem 18. Let P be the part of the graph $z = x^2+y$ that lies above the square $1 \le x \le 2$ and $0 \le y \le 1$ in the xy-plane, with normal pointing upwards.

a. Give a parametrization of P with the specified normal.

b. Find the flux of the vector field $\mathbf{F}(x,y,z) = -x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$, through P.

Problem 19. For the given integral $\int_0^2 \int_{y/2}^1 e^{-x^2}\, dx\, dy$:

a. Sketch the domain of the integral.
b. Reverse the order of integration.
c. Find the value of this integral.

Problem 20. Sketch the region D defined by $x^2 + y^2 \leq 4$, $x \geq 0$ and $y \geq 0$ in $\mathbf{R}^2$, describe the region D by using polar coordinates, and calculate $\iint_D x^2\sqrt{x^2+y^2}\, dx\, dy$.

Problem 21. Let B be the region inside the sphere $x^2+y^2+z^2 = 8$ and above the cone $z = \sqrt{x^2+y^2}$ in $\mathbf{R}^3$. Set up iterated integrals for the triple integral $\int_B x^2+y^2\, dV$ in the following coordinates.

DO NOT EVALUATE THE INTEGRALS.

a. Rectangular (x,y,z) coordinates
b. Cylindrical coordinates
c. Spherical coordinates

Problem 22. Let a transformation T from the uv-plane to the xy-plane be defined by $x = u+v$ and $y = u-v$. Let R_{uv} be the rectangular region given by $0 \leq u \leq 2$, and $0 \leq v \leq 1$ in the uv-plane.

a. Find and sketch the region $R_{xy} = T(R_{uv})$, the image of R_{uv} under the transformation T.
b. Find $\frac{\partial(x,y)}{\partial(u,v)}$.
c. Transform $\int_{R_{xy}} xy\, dx\, dy$ to an integral over R_{uv}. Write the integral as an iterated integral, but do not evaluate it.

Problem 23. For the curve L parametrized by $g(t) = (2t, \ln t, t^2)$, for $1 \leq t \leq e$, calculate the following.

a. The length of L.
b. The mass of a wire along the curve L with density z at a point (x,y,z).
c. The line integral $\int_L x\, dx + (x^2+z)dy$

Problem 24. **a**. Is the vector field $\mathbf{F}(x,y) = (y\cos xy,\ 2+x\cos xy) : \mathbf{R}^2 \to \mathbf{R}^2$ conservative? Explain why if it is not. Find a potential function for it, if it is conservative.

b. By using part (a), calculate the line integral $\int_C (y\cos xy)dx + (2+x\cos xy)dy$, where C is the curve given by $g(t) = (\pi(1-t)^3,\ e^{2t}) : [0,1] \to \mathbf{R}^3$.

Problem 25. Find $\int_C (e^{x^2}-y)dx + (e^{y^2}+2x)dy$, where C is boundary of the square with vertices $(0,0)$, $(3,0)$, $(3,3)$ and $(0,3)$ in the xy –plane traversed counterclockwise.