1/17/18

(1)

(I) HW to read Chap I

(1.3)
- $x \in A$, $x \notin A$
- $\emptyset$
- $A \subset B$ same as $A \subseteq B$

  same as $A \subseteqq B$

- Proper subset B of A $\begin{cases} B \subset A \\ B \neq A \end{cases}$ and

Notation: $B \subsetneqq A$

- $A \notin A$, $1 \in \{1\}$ } true; $A \subseteq A$, $1 \not\subseteq \{1\}$ } all true | False: $A \in A$; False: $1 \subseteq \{1\}$

$\forall$, $\exists$ notation

(1.4)
$$\mathbb{N} = \{1, 2, 3, \ldots\}$$
$$\mathbb{Z} = \{0, 1, -1, 2, -2, 3, -3, \ldots\}$$
$$\mathbb{Q} = \{ p/q \mid p, q \in \mathbb{Z}, q \neq 0 \}$$

Relation: on a set S is a subset T of $S \times S$

Ex $\{(x,y) \mid x^2 = y\} \subseteq \mathbb{R} \times \mathbb{R}$

is a relation on $\mathbb{R}$.

Defn A relation T on S is called an equivalence relation if

(i) T is reflexive ($\forall a \in S$, $(a,a) \in T$) and

(ii) T is symmetric ($\forall a,b$ $(a,b) \in T \Rightarrow (b,a) \in T$) and

(iii) T is transitive:

$$\forall a,b,c \in S \ \left((a,b) \in T, (b,c) \in T \Rightarrow (a,c) \in T \right)$$

Order relation $<$ on S

$$a < b \iff (a,b) \in T \text{ order relation}$$

satisfies

(i) Trichotomy

$\forall x,y \in S$, one and only one of the following hold:

$$x < y \quad \text{or} \quad x = y \quad \text{or} \quad y < x.$$

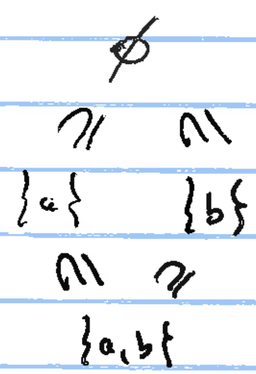
(ii) $<$ is transitive.

$$\forall x,y,z \in S \ (x < y \ \& \ y < z \Rightarrow x < z)$$

Caution "order relation" depends on the book/source.

Partial orders: (No trichotomy)

Ex. $a \neq b$.



$\{a\} \not\subseteq \{b\}$

$\{a\} \not\supseteq \{b\}$

$\{a\} \neq \{b\}$

"$\subseteq$" is a partial order.

Notation: $x \leq y \iff (x<y \text{ or } x=y)$

(1.7) Let S be an ordered set with $<$.

- Let $E \subseteq S$. E is called bounded above if $\exists \beta \in S$ s.t. $\forall x \in E$ $x \leq \beta$.

- Let $E \subseteq S$. E is called bounded below, if $\exists \gamma \in S$ s.t. $\forall x \in E$ $x \geq \gamma$.

(1.8) Let S be an ordered set, let E be bounded above.
If $\exists \alpha \in S$ s.t.
(i) α is an upper bound for E $(\forall x \in E \quad x \leq \alpha)$, and
(ii) If $\gamma < \alpha$, then γ is not an upper bound for E
(i.e. $\text{not}(\forall x \in E,\ x \leq \gamma) \equiv \exists x \in E \quad x > \gamma$)
then α is called the least upper bound of E
or the supremum of E.

Notation:
$\alpha = \sup E = \text{lub}\, E$

Uniqueness follows from trichotomy i.e. if there were two $\alpha_1, \alpha_2 \in S$ which satisfy (i) & (ii), one would get a contradiction from $\alpha_1 < \alpha_2$
" " " " " $\alpha_2 < \alpha_1$

which only leaves $\alpha_1 = \alpha_2$ option.

Similarly Let B be a set bounded below. If $\exists \beta \in S$ s.t. (i) $\forall x \in B$, $x \geq \beta$, and
Caution type (ii) If $\gamma > \beta$, then γ is not a lower bd for B.
then $\beta = \inf B = \text{glb}\, B$.