**MATH 3770, Section C, Fall 2019, Review for Final Exam**

The final exam is comprehensive. You are responsible for all of the material covered in this course since the beginning of the semester, regardless of what is listed in this review. Most of the proof type questions will be chosen from sections of 5.1, 5.2, 5.3, 6.1, 6.2 and lecture notes (November 1, 4, 6) on subsequences and compactness. Some (parts of) questions may involve methods, proofs and topics from the earlier parts of the course. True/False questions, statement and definition type questions **will** be from all parts of the course material, including the earlier parts of the course. The questions about applications, examples, HW may be from all chapters 4-6.

Your preparation for the final exam should include the following.

1. Learn all correct solutions of the assigned HW problems, even if some of them are not to be handed in.
2. Learn all examples and exercises done in class.
3. Learn all proofs done in class unless they are specifically excluded below in sections. You may need to reproduce some of the proofs. Make sure to learn all the proofs specified with stars (\*)-(\*\*\*\*\*) in the class notes.
4. Learn all definitions, since you need to know them to do problems in the test.
5. Learn the statements of the theorems precisely. Especially, you may be asked to state some named theorems, definitions and axioms in the test.
6. Do not expect the questions to be identical to those in 1-6 above; there can be some differences.
7. The old tests are available in the course web page from 2018. They may give you an idea about your test. However, they are tests for a similar but another class given during another semester. You should not expect an identical test. There will be differences between your test and the old tests. For that reason, your preparation must go beyond studying the old tests.

**Section 4.4 Subsequences (Lecture notes from November 1)**

* Definition 4.4.1 of subsequences
* Theorem 4.4.4 and its converse, statement and proofs.
* Theorem 4.4.7 statement, but not its proof (actually given later on page 7 of Nov 20 notes).
* We did not cover 4.4.8-15.

**Section on Compactness (Lecture notes from November 4 and 6)**

* Definition of compactness and Heine-Borel Theorem (statement)
* Statements of Theorems on page 5 of Nov. 4 lecture notes
* The Bolzano-Weierstrass Theorem and its proof as it is done in class, on Nov 4. (The proof in the book uses open covers, which we did not study.)
* Rest of the Nov. 4 lecture.
* Learn how to identify compact sets in real numbers (or rational numbers) on examples, with proofs if needed, Nov. 6 notes.

**Section 5.1**

* (e-d) Definition 5.1.1 of limits of functions,
* **Must** be able to carry out specific limit examples by the use of (e-d) definition similar to Examples 5.1.5, 5.1.6, and HW (see Nov 11 notes)
* Sequential criterion for limits, Theorems 5.1.8 and 5.1.10, Statements and proofs
* To be able to apply these theorems for showing the non-existence of limits, such as Example 5.1.11, and HW
* Theorem 5.1.13 Statement, proof and applications

**Section 5.2**

* (e-d) Definition 5.2.1 of continuity of functions
* To be able to carry out specific continuity examples by the use of (e-d) definition, such as Exercise 5.2.4
* Theorem 5.2.2, Statement and proof
* Examples 5.2.7, 8 and examples done in class (Nov 18).
* Theorem 5.2.10, Statement and proof
* Theorem 5.2.12 about the continuity of compositions, Statement and proof (learn the two different proofs done in class, Nov 18, pages 5, 6)
* We did not cover Theorems 5.2.14-16 as they are done in the book.

**Section 5.3**

* The proof of Theorem 5.3.2 in the book uses open covers which we did not discuss.
* Learn the following:
1. Statement of Theorem 5.3.2 for closed and bounded subsets D of real numbers, and learn its proof as well as Lemma I, both given in class on Nov. 20, pages 4-6
2. Learn all of the notes from Nov. 20 (page 7 proof of I implies II excluded)
3. If Ais a closed and bounded subset of **R** then ithas max and min, learn proof (Lemma II).
4. **Extreme Value Theorem** with statement and proof, Corollary 5.3.3
5. Examples showing something goes wrong when some of the hypothesis is dropped inthe Extreme value theorem.
6. **Intermediate Value Theorem** with Statement, proof (5.3.5 and 5.3.6) (Nov 22, pages 4-7) andapplications
7. Theorem 5.3.10, Statement and proof (Dec 2)
8. Redo Exercise 5.3.3 carefully (Dec 2)

**Section 6.1**

* Definition 6.1.1. of derivative,
* To be able to carry out specific derivative examples by the use of definition, such as Exercises 6.1.4, 6, 7d and all examples done in class Dec 4.
* Theorem 6.1.3, sequential criterion
* To be able to carry out specific non-differentiability examples by the use of Theorem 6.1.3, such as Example 6.1.4, and examples done in class Dec 4.
* Must know Theorem 6.1.6, statement and proof (differentiability implies continuity)
* Must know Theorem 6.1.7, statement and proofs (Algebra of derivatives, including quotient rule)
* Must know Theorem 6.1.10 **Chain Rule**, with statement and proof
* Learn Exercise 6.1.15 (Dec 6)

**Section 6.2**

* Theorem 6.2.1, statement and proof (**First Derivative Test**)
* Theorem 6.2.2, statement and proof (**Rolle's Theorem**)
* Theorem 6.2.3, statement and proof (**Mean Value Theorem**)

There are MANY applications of **Mean Value Theorem,** for example Dec 11 notes as well as many HW problems and exercises from 6.2.

* Learn the statements and proofs of Theorems: 6.2.6, 7, 8, as well as Exercises 6.2.8 and 6.2.9 as they are done in class.
* We did not cover Theorems 6.2.9-6.2.12, pp 252-254