MATH 3770, Section A, Fall 2018, Review for Final Exam

The final exam is comprehensive. You are responsible for all of the material covered in this course since the beginning of the semester, regardless of what is listed in this review. Most of the proof type questions will be chosen from sections of 4.4, 5.1, 5.2, 5.3, 6.1, 6.2 and lecture notes on compactness. Some (parts of) questions may involve methods, proofs and topics from the earlier parts of the course. True/False questions **will** have statements from all parts of the material, including the earlier parts of the course as well as Chapter 7. See page 3. Of course, the questions about applications, examples, HW can be from all chapters 4-7.

Your preparation for the final exam should include the following.

1. Learn all correct solutions of the assigned HW problems, even if some of them are not to be handed in.
2. Learn all examples and exercises done in class.
3. Learn all proofs done in class unless they are specifically excluded below in sections. You may need to reproduce some of the proofs. Make sure to learn all the proofs specified with stars (*)-(*****) in the class notes.
4. Learn all definitions, since you need to know them to do problems in the test.
5. Learn the statements of the theorems precisely. Especially, you may be asked to state some named theorems, definitions and axioms in the test.
6. Do not expect the questions to be identical to those in 1-6 above; there can be some differences.
7. The old tests are available in the course web page. They may give you an idea about your test. However, they are tests for a similar but another class given during another semester. You should not expect an identical test. There will be differences between your test and the old tests. For that reason, your preparation must go beyond studying the old tests.

Section 4.4

- Definition 4.4.1 of subsequences
- Theorem 4.4.4 and its converse, statement and proofs.
- Theorem 4.4.7 statement, but not its proof.
- We did not cover 4.4.8-15.

Section on Compactness (Lecture notes from October 29 and 31)

- Definition of compactness and Heine-Borel Theorem (statement)
- Definition of sequentially compact.
- Statements of Theorems 1-3 on page 5 of Oct. 29 lecture notes
- The Bolzano-Weierstrass Theorem and its proof as it is done in class, on Oct. 31. (The proof in the book uses open covers, which we did not study.)
- Rest of the Oct. 31 lecture.
- Learn how to identify compact sets in real numbers on examples, with proofs if needed.

Section 5.1

- (ε–δ) Definition 5.1.1 of limits of functions,
- **Must** be able to carry out specific limit examples by the use of (ε–δ) definition such as Examples 5.1.5, 5.1.6, and HW
- Sequential criterion for limits, Theorems 5.1.8 and 5.1.10, Statements and proofs
- To be able to apply these theorems for showing the non-existence of limits, such as Example 5.1.11, and HW
- Theorem 5.1.13 Statement, and proof

Section 5.2

- (ε–δ) Definition 5.2.1 of continuity of functions
- To be able to carry out specific continuity examples by the use of (ε–δ) definition, such as Exercise 5.2.4
- Examples 5.2.7, 8 and examples done in class.
- Theorem 5.2.10, Statement and proof
- Theorem 5.2.12, Statement and proof (learn the two different proofs done in class)
- We did not cover Theorems 5.2.14-16 as they are done in the book.
- However, we proved a theorem about continuity and pre-images of closed or open sets on Nov. 12, p 4-6. Learn those proofs.

Section 5.3

- The proof of Theorem 5.3.2 in the book uses open covers which we did not discuss.
- Learn the following:

1. Statement of Theorem 5.3.2 for closed and bounded subsets D of real numbers, and learn its proof as well as Lemma I and its proof both given in class on Nov. 14
2. If A is a closed and bounded subset of **R** then it has max and min, with proof.
3. **Extreme Value Theorem** with statement and proof, Corollary 5.3.3
4. Examples showing something goes wrong when some of the hypothesis is dropped in the Extreme value theorem.
5. **Intermediate value theorem** with Statement, proof (5.3.5 and 5.3.6) and applications
   Theorem 5.3.10, Statement and proof
   Do Exercise 5.3.3 carefully (in the homework)

Section 6.1

- Definition of derivative,
- To be able to carry out specific derivative examples by the use of definition, such as Exercises 6.1.4, 7d, 8 (done in class)
- Theorem 6.1.3, sequential criterion
- To be able to carry out specific non-differentiability examples by the use of Theorem 6.1.3, such as Example 6.1.4
- Must know Theorem 6.1.6, statement and proof (differentiability implies continuity)
- Must know Theorem 6.1.7, statement and proofs (Algebra of derivatives)
- Must know Theorem 6.1.10 **Chain Rule**, with statement and proof

Section 6.2

- Theorem 6.2.1, statement and proof (**First derivative test**)
- Theorem 6.2.2, statement and proof (**Rolle's theorem**)
- Theorem 6.2.3, statement and proof (**Mean value theorem**)
  There are MANY applications of **Mean value theorem,** for example Dec 3 notes as well as many HW problems.
- Learn the statements and proofs of Theorems: 6.2.6, 7, 8
- We did not cover Theorems 6.2.9-6.2.12, pp 252-254

Since we covered the sections 7.1-3 late in the semester, the final exam questions from those sections are going to be limited to applications, examples and the recommended exercises (with possible changes of the rewording, functions and the numbers). You need to know the statements of theorems discussed in class, but not their proofs as given on Dec. 5 & 7. The following is the list of what we covered during the last week, with the references to the textbook.

Sections 7.1, 2

- Definitions 7.1.1-3
- Statements (no proofs) of Theorems 7.1.4, 7.1.6, 7.1.9
- Example 7.1.8
- Learn the statement of Theorem 7.2.2 (Continuous on [a, b] implies integrable over [a, b]). Understanding this theorem requires to know the following important points:
  - Definition of Uniform Continuity p 222
  - Statement (no proof) of Theorem 5.4.6, p 224: Every continuous function from a (compact) closed and bounded set in **R** is uniformly continuous.

Section 7.3 Fundamental Theorems I-II of Calculus

- Learn the statement of Theorem 7.3.1 p 293, and applications
- Learn the statement of Theorem 7.3.5 p 295, and applications
- Corollary and examples 7.3.2-4, and examples and exercises done in class.
- Recommended HW Exercises (even though they are not to be handed in):
  7.3 # 5, 6cd, 10