MATH 2850

Final

Fall 2016

Problem 1. Find an equation (of the form $Ax + By + Cz = D$) for the plane tangent to the surface

$$x = e^s$$
$$y = t^2 e^{2s}$$
$$z = e^{-s} + t$$

at the point $(1, 1, 0)$.

Problem 2. Find the flux $\iint_S \mathbf{F} \cdot d\mathbf{S}$ of $\mathbf{F}\,(x, y, z) = 2x\,\mathbf{i} + y\,\mathbf{j} + z\,\mathbf{k}$ across the surface S consisting of the triangular region of the plane $2x + 2y + z = 2$ that is cut out by the coordinate planes. Use the upward-pointing normal to orient S.

Problem 3. Verify the Stokes's Theorem (by computing both integrals and showing that they are equal) for the surface S defined by $x^2 + y^2 + z = 1$, $z \geq 0$, oriented by upward normal, and the vector field $\mathbf{F}\,(x, y, z) = xz\,\mathbf{i} + yz\,\mathbf{j} + (x^2 + y^2)\,\mathbf{k}$.

Problem 4.

a. Determine whether $\mathbf{F}(x, y, z) = (2x + y)\,\mathbf{i} + (x + z\cos yz)\,\mathbf{j} + (y\cos yz)\,\mathbf{k}$ is a conservative vector field. If it is, find a scalar potential function for $\mathbf{F}$. If it is not, explain why.

b. Calculate the line integral $\int_C \mathbf{F} \cdot d\mathbf{s}$, where C is the curve given by $\mathbf{x}(t) = (t, \cos t, \sin t) : [0, \frac{\pi}{2}] \to \mathbf{R}^3$.

Problem 5. Calculate $\oint_C (\sin(x^2) + e^y)dx + (x^2 y - e^{y^2})dy$, where C is the path formed by following the edges of the rectangle with vertices $(0, 0)$, $(1, 0)$, $(1, 2)$, and $(0, 2)$ oriented clockwise.

Problem 6. a. Give a parametrization of the curve C obtained by intersecting the plane $z = 2y$ and the cylinder $x^2 + y^2 = 4$ and oriented counterclockwise around the $z-$axis as seen from the positive $z-$axis. Sketch the curve C.

b. Calculate the line integral $\int_C \mathbf{F} \cdot d\mathbf{s}$ along the curve C of part (**a**), where $\mathbf{F}\,(x, y, z) = 3z\,\mathbf{i} + 2x\,\mathbf{j} + y\,\mathbf{k}$.

Problem 7. Evaluate the integral $\int_0^1 \int_{x/2}^{(x/2)+1} x^4 (2y - x) e^{(2y-x)^2}\,dy dx$ by making the substitution $u = 2y - x$ and $v = x$.

Problem 8. Let $\iiint_W (x^2+y^2+z^2)dV$ be given where W is the region inside the sphere $x^2+y^2+z^2=1$ and above the cone $z=\sqrt{x^2+y^2}$.

a. Sketch the domain of integration W.

b. Express this integral in spherical coordinates as an iterated integral with bounds.

c. Express this integral in cylindrical coordinates as an iterated integral with bounds.

d. Compute the value of this integral.

Problem 9.

a. Evaluate $\int_{-a}^{a}\int_{-\sqrt{a^2-x^2}}^{\sqrt{a^2-x^2}} 4y^2\ dy\ dx$

b. Evaluate $\int_0^1\int_y^1 x^2 \sin xy\ dx\ dy$

Problem 10.

Let $\mathbf{F}(x,y,z)=x\ \mathbf{i}+y\ \mathbf{j}+(3-2z)\ \mathbf{k}$, and let S be defined by $z=e^{1-x^2-y^2}$, $z\geq 2$, oriented upward normal. Calculate $\iint_S \mathbf{F}\cdot d\mathbf{S}$.(HINT: What is the divergence of F?)