22M28 Midterm 1 Practice - 100 minutes

1. Let $P = (1, 1, 2)$, $Q = (2, 0, 1)$ and $R = (1, 0, 0)$.

a. Find a parametric representation for the line passing through P and Q.

b. Find a closed equation (i.e. $Ax + By + Cz = D$) for the plane passing through P, Q and R.

c. Find the area of the triangle ΔPQR.

2. Let $g(x, y) = 4 - x^2 - y^2 : \mathbf{R}^2 \to \mathbf{R}$.

a. Sketch the level sets of g for the values $k = 0, 4, 8$. Label each axis.

b. Sketch the (explicit) graph of g. Label each axis.

3. Let $h(x, y) = y^2x^3 + e^{3x-y}$.

a. Calculate all first and second order partial derivatives of h.

b. Find an equation describing the tangent plane to the explicit graph of $z = h(x, y)$ when $x = 1$ and $y = 3$.

4. Let $f(x, y, z) = (x^2 - y^2, xyz)$ and $g(u, v) = (uv, u - 3v, u^2 + v^3)$.
Calculate the following: $Df, Dg, D(f \circ g)(1, 2)$ and $D(g \circ f)(1, 2, 3)$

5. Let $f(x, y) = x^2 - \frac{y}{2}$. **Label all axes. Label all curves with the appropriate values to distinguish them.**

a. Sketch the sections $z = f(a, y)$, in the planes $x = a$ for $a = 0$ and 1.

b. Sketch the sections $z = f(x, b)$, in the planes $y = b$ for $b = 0$ and 1.

c. Sketch the contour curves (implicit graphs) $f(x, y) = c$ for $c = 0$ and 1.

d. Sketch the explicit graph of $z = f(x, y)$. Describe it in words if you can't draw it.

6. Let $P = (1, 4, 3)$, $Q = (3, 5, 4)$ and $R = (-1, 6, 5)$ in $\mathbf{R}^3$.

a. Find the angle $\measuredangle QPR$ at the vertex P of the triangle $\triangle PQR$.

b. Find a parametric equation for the plane passing through the points P, Q and R.

7. a. Calculate the following: $\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}, \frac{\partial x}{\partial u}, \frac{\partial x}{\partial v}, \frac{\partial y}{\partial u}, \frac{\partial y}{\partial v}, \frac{\partial z}{\partial u}, \frac{\partial z}{\partial v}$ where $\left\{ \begin{array}{rcl} z & = & \sin(xy) \\ x & = & 2ue^{3v} \\ y & = & vu^4 + \ln u \end{array} \right.$.

b. Let $f(x, y) = \sin(xy)$ and $g(u, v) = (2ue^{3v}, vu^4 + \ln u)$, and $h = f \circ g$. Calculate the derivative $Dh(1, 0)$.

8. Let $g(x, y) = xe^{-y}$.

a. Calculate the gradient ∇g and the derivative Dg of g.

b. Find the (explicit) tangent plane to the (explicit) graph of $z = g(x, y)$ above $(x, y) = (3, 0)$.

c. Find the approximate value of $g(3.1, 0.3)$ by using (a) or (b) above.
(The value 2.296536484... which you may read from a calculator, is NOT the answer.)

d. In which direction does g increase at the fastest rate at $(x, y) = (3, 0)$?
What is the steepest rate of increase at $(x, y) = (3, 0)$?

9. Let $F(x, y, z) = x^3 + 3xyz - 2z^2$

a. Calculate the gradient of F.

b. Find the directional derivative of F at the point $(2, 0, -1)$ in the direction of the vector $(2, 1, -2)$.

c. Find an equation for the tangent plane to the surface $F(x, y, z) = 6$ at $(2, 0, -1)$.

d. It is given that the equation $x^3 + 3xyz - 2z^2 = 6$ defines x implicitly in terms of y and z near $(2, 0, -1)$. Calculate $\frac{\partial x}{\partial z}(2, 0, -1)$.

10. Write the final answer only.

a. Let $A = \{(x, y)|0 < x < 1 \text{ and } 0 \leq y \leq 2\}$.
What is the boundary of A?
Is A an open set? Circle the correct answer: YES or NO.
Is A a closed set? Circle the correct answer: YES or NO

b. Calculate the following limits. If any of them does not exist, state it so.

i. $\lim_{t \to 2} (t^2 + 2t, \frac{t^2-1}{t-1}, e^t) =$

ii. $\lim_{(x,y) \to (0,0)} \frac{x^2}{x^2+y^2} =$

iii. $\lim_{(x,y) \to (1,1)} \frac{xy-x}{1-y^2} =$