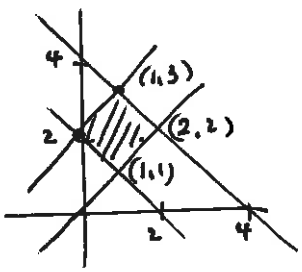
MATH 2850
Quiz 5
April 27, 2017

SOLUTION

NAME.______________________________

1. Determine the value of $\iint_D (x+y)\, e^{(y^2-x^2)}\, dA$ where D is the region enclosed by: $\begin{cases} x+y=2 \\ x+y=4 \\ y=x \\ y=x+2 \end{cases}$



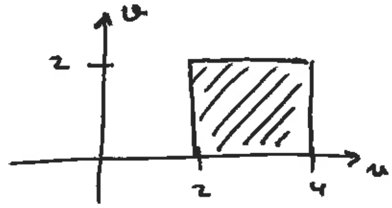
$T(x,y) = (x+y,\ y-x) = (u,v)$ Linear substitution

$T(1,1) = (2,0)$

$T(2,2) = (4,0)$

$T(0,2) = (2,2)$

$T(1,3) = (4,2)$



$u = x+y$

$v = y-x$

$\dfrac{\partial(u,v)}{\partial(x,y)} = \begin{vmatrix} 1 & 1 \\ -1 & 1 \end{vmatrix} = 2$

$2\,dx\,dy = du\,dv$

$dx\,dy = \frac{1}{2}\,du\,dv$

$$\iint_D (x+y)\, e^{y^2-x^2}\, dx\,dy = \int_2^4 \int_0^2 \frac{1}{2}\cdot u \cdot e^{uv}\, dv\, du = \int_2^4 \frac{1}{2} e^{uv} \Big|_{v=0}^{v=2} du$$

$$= \int_2^4 \frac{1}{2}\left(e^{2u} - e^0\right) du = \frac{1}{2}\left(\frac{1}{2}e^{2u} - u\right)\Big|_{u=2}^{u=4}$$

$$= \frac{1}{2}\left(\frac{1}{2}e^8 - 4 - \frac{1}{2}e^4 + 2\right) = \frac{1}{4}\left(e^8 - e^4\right) - 1.$$

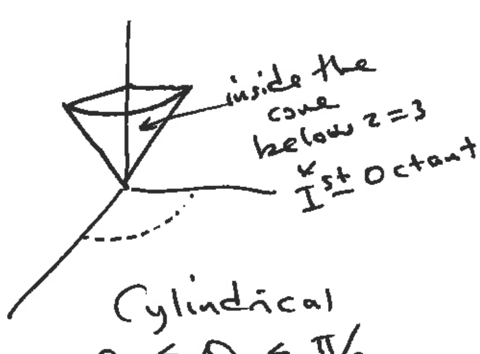
PTO for Alternate substitution

2. Evaluate the following integral by using cylindrical coordinates. Sketch the domain of the integration in Cartesian coordinates.

$0 \le y \le 3$

$0 \le x \le \sqrt{9-x^2}$

$0 \le z \le \sqrt{x^2+y^2}$



Cylindrical

$0 \le \theta \le \pi/2$

$0 \le r \le 3$

$0 \le z \le r \left(= \sqrt{x^2+y^2}\right)$

$$\int_0^3 \int_0^{\sqrt{9-y^2}} \int_{\sqrt{x^2+y^2}}^3 x\, dz\, dx\, dy$$

Jacobian $\dfrac{\partial(x,y,z)}{\partial(r,\theta,z)}$

$$= \int_0^{\pi/2} \int_0^3 \int_r^3 \underbrace{r\cos\theta}_{x} \cdot r \cdot dz\, dr\, d\theta$$

$$= \int_0^{\pi/2} \int_0^3 \cos\theta \cdot r^2 \cdot z \Big|_{z=r}^{z=3} dr\, d\theta$$

$$= \int_0^{\pi/2} \int_0^3 \cos\theta \left(3r^2 - r^3\right) dr\, d\theta$$

$$= \sin\theta \Big|_0^{\pi/2} \cdot \left. r^3 - \frac{r^4}{4} \right|_0^3 = 1 \cdot \left(27 - \frac{81}{4}\right)$$

$$= 27\left(1 - \frac{3}{4}\right) = \frac{27}{4}.$$

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NAME. Alternate Sol^n

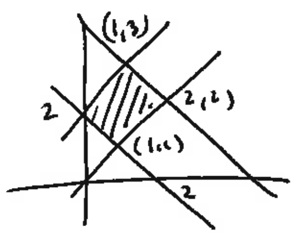
1. Determine the value of $\iint_D (x+y)\, e^{(y^2-x^2)}\, dA$ where D is the region enclosed by: $\begin{cases} x+y=2 \\ x+y=4 \\ y=x \\ y=x+2 \end{cases}$

$u = x+y$

$v = y^2 - x^2$

$$\frac{\partial(u,v)}{\partial(x,y)} = \begin{vmatrix} 1 & 1 \\ -2x & 2y \end{vmatrix} = 2(y+x) = 2u$$

$T(x,y) = (x+y,\ y^2-x^2)$ Non-linear

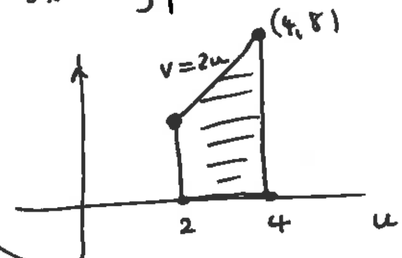


$T(1,1) = (2,0)$

$T(0,2) = (2,4)$

$T(2,2) = (4,0)$

$T(1,3) = (4,8)$



$x+y=2 \to u=2$

$x+y=4 \to u=4$

$y=x \to v=0.$

required (Not obvious)

$y = x+2$ goes to: $T(t, t+2)$ $\quad 0 \le t \le 1$

$= (2t+2,\ (t+2)^2 - t^2)$

$= \underbrace{(2t+2,\ 4t+4)}_{}$

$v = 2u$ line between $(2,4)$ & $(4,8)$

$$\boxed{\begin{aligned} 2(x+y)\,dx\,dy &= du\,dv \\ (x+y)\,dx\,dy &= \tfrac{1}{2}\,du\,dv \end{aligned}}$$

$$\int_2^4 \int_0^{2u} e^v \,\frac{1}{2}\, dv\, du = \int_2^4 \frac{1}{2} e^v \Big|_0^{2u} = \int_2^4 \frac{1}{2}(e^{2u}-1)\, dv$$

$$= \frac{1}{2}\left(\frac{1}{2}e^{2u} - u\right)\Big|_2^4 = \frac{1}{2}\left[\frac{1}{2}e^8 - \frac{1}{2}e^4 - 2\right] = \frac{1}{4}(e^8 - e^4) - 1$$

2. Evaluate the following integral by using cylindrical coordinates. Sketch the domain of the integration in Cartesian coordinates.

$$\int_0^3 \int_0^{\sqrt{9-y^2}} \int_{\sqrt{x^2+y^2}}^{3} x\, dz\, dx\, dy$$