MATH 2850
Quiz 4
April 6, 2017

NAME. SOLUTION

1. Let $F(x,y) = x^3 - 2xy - x + y^2$. Find all critical points of F, and determine the nature of each (local max, min, saddle, or degenerate).

$F_x = 3x^2 - 2y - 1$

$F_y = -2x + 2y$

Solve ① $3x^2 - 2y - 1 = 0$

② $-2x + 2y = 0$

② $\Rightarrow$ $x = y$

① $3x^2 - 2x - 1 = 0$

$x = \dfrac{2 \pm \sqrt{4+12}}{6} = \dfrac{2 \pm 4}{6} = 1, -\frac{1}{3}$

$x = y \Rightarrow$ c.p. $(1,1)$, $(-\frac{1}{3}, -\frac{1}{3})$

$H_F = \begin{bmatrix} 6x & -2 \\ -2 & 2 \end{bmatrix}$.

$H_F(1,1) = \begin{bmatrix} 6 & -2 \\ -2 & 2 \end{bmatrix}$ $\Delta_1 = 6 > 0$, $\Delta_2 = 8 > 0$

Positive definite, Local min at $(1,1)$

$H_F(-\frac{1}{3}, -\frac{1}{3}) = \begin{bmatrix} -2 & -2 \\ -2 & 2 \end{bmatrix}$

$\Delta_1 = -2$

$\Delta_2 = -8 < 0$, $n = 2$ $\Rightarrow$ indefi

$\Rightarrow$ saddle at $(-\frac{1}{3}, -\frac{1}{3})$

2. Find the maximum and the minimum values of $f(x,y) = x + 2y$ subject to the constraint $2x^2 + y^2 = 18$. If either the maximum or the minimum value does not exist, state it so.

$\nabla f = (1, 2)$

$g = 2x^2 + y^2 = 18$

$\nabla g = (4x, 2y)$

$\nabla f = \lambda \nabla g \Rightarrow \begin{cases} 1 = 4\lambda x & ① \\ 2 = 2\lambda y & ② \end{cases}$

constraint: $2x^2 + y^2 = 18$ ③

① $\Rightarrow$ $\lambda = \dfrac{1}{4x}$ ($x \neq 0$ since ①)

② $\Rightarrow$ $\lambda = \dfrac{1}{y}$ ($y \neq 0$ since ②)

$\dfrac{1}{4x} = \dfrac{1}{y} \Rightarrow y = 4x$

③ $\Rightarrow$ $2x^2 + y^2 = 2x^2 + 16x^2 = 18$

$18x^2 = 18$

$18x^2 = 18 \Rightarrow x = \pm 1$

$y = 4x$

c.p. $(1, 4)$, $(-1, 4)$

	$f = x + 2y$	
$(1,4)$	9	largest
$(-1,-4)$	-9	smallest

Since $2x^2 + y^2 = 18$ is an ellipse, it is bounded & closed, hence compact. f is continuous. $\exists$ max and min values. $9 = $ max value, $-9 = $ min value.