MATH 2850
Quiz 3
March 23, 2017 NAME.______________________________

1. The equations $\begin{cases} x + 2y + e^{2z} = 3 \\ x + x\cos z + ze^{y} = 0 \end{cases}$ define y and z in terms of x by $h(x) = (y,z)$ near $(x,y,z) = (0,1,0)$. Find the derivative matrix $h'(0)$. What is $\frac{\partial y}{\partial x}(0)$?

$$F(x,y,z) = (x + 2y + e^{2z},\ x + x\cos z + ze^{y})$$

$$DF = \begin{bmatrix} 1 & 2 & 2e^{2z} \\ 1+\cos z & ze^{y} & -x\sin z + e^{y} \end{bmatrix}$$

$$DF(0,1,0) = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 0 & e \end{bmatrix}.$$ Since $\begin{vmatrix} 2 & 2 \\ 0 & e \end{vmatrix} = 2e \neq 0$, by Implizit Func. Thm. $\exists\ h(x) = (y,z)$ locally near $(x,y,z) = (0,1,0)$.

$$h'(0) = -\begin{bmatrix} 2 & 2 \\ 0 & e \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = -\frac{1}{2e}\begin{bmatrix} e & -2 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \end{bmatrix} = \frac{-1}{2e}\begin{bmatrix} e-4 \\ 4 \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{1}{2} + \frac{2}{e} \\ -\frac{2}{e} \end{bmatrix}; \qquad \frac{\partial y}{\partial x}(0) = \frac{4-e}{2e} = -\frac{1}{2} + \frac{2}{e}.$$

2. Let $\mathbf{x}(t) = (\cos 2t, \sin 2t, \frac{2}{3}t^{\frac{3}{2}})$ for $0 \le t \le 5$. Calculate the following for the curve $\mathbf{x}(t)$. Indicate which is which.

i. The velocity $\longrightarrow x'(t) = (-2\sin 2t,\ 2\cos 2t,\ t^{1/2})$

ii. The acceleration $\rightarrow x''(t) = (-4\cos 2t,\ -4\sin 2t,\ \frac{1}{2}t^{-\frac{1}{2}})$

iii. The speed $\longrightarrow \|x'(t)\| = \sqrt{4\sin^2 2t + 4\cos^2 2t + t} = \sqrt{4+t}$

iv. The length.

v. An equation of the tangent line to $\mathbf{x}(t)$ when $t = 4$

$$\int_0^5 \sqrt{4+t}\,dt = \int_4^9 u^{1/2}\,du = \frac{2}{3}u^{3/2}\Big|_4^9 = \frac{2}{3}\left(9^{\frac{3}{2}} - 4^{\frac{3}{2}}\right) = \frac{2}{3}(27-8) = \frac{38}{3}.$$

$u = 4+t$
$du = dt$

$$\ell(s) = x(4) + x'(4)\cdot s$$
$$= (\cos 8,\ \sin 8,\ \tfrac{2}{3}4^{3/2}) + s(-2\sin 8,\ 2\cos 8,\ 4^{1/2})$$
$$= (\cos 8,\ \sin 8,\ \tfrac{16}{3}) + s(-2\sin 8,\ 2\cos 8,\ 2)$$