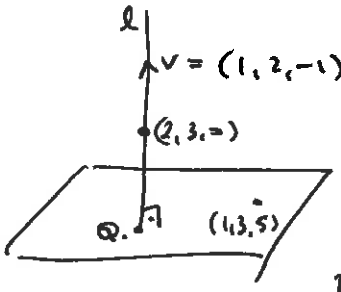
MATH 2850
Quiz 2
February 16, 2017

NAME. SOLUTION

1. a. Find an equation of the form $Ax + By + cz = D$ for the plane passing through the point $P = (1,3,5)$ and perpendicular to the line ℓ : $\begin{cases} x = 2+t \\ y = 3+2t \\ z = -t \end{cases}$



$v \perp$ plane

$(1,2,-1) \cdot ((x,y,z) - (1,3,5)) = 0$

$x + 2y - z = (1,2,-1) \cdot (1,3,5)$

$= 1 + 6 - 5 = 2$

$\boxed{x + 2y - z = 2}$

b. What is the distance between the point P and the line ℓ (from part **a**)?

METHOD I Q, the intersection of the plane & line ℓ is the closest pt ℓ to (1,3,5) since $\ell \perp$ plane.

$x + 2y - z = 2$

$x = 2 + t$

$y = 3 + 2t$

$z = -t$

$(2+t) + 2(3+2t) - (-t) = 2$

$8 + 6t = 2$

$6t = -6$

$t = -1$

$Q = (1, 1, 1)$

$dist = \|P - Q\| = \|(0, 2, 4)\| = \sqrt{20} = 2\sqrt{5}.$

METHOD II →

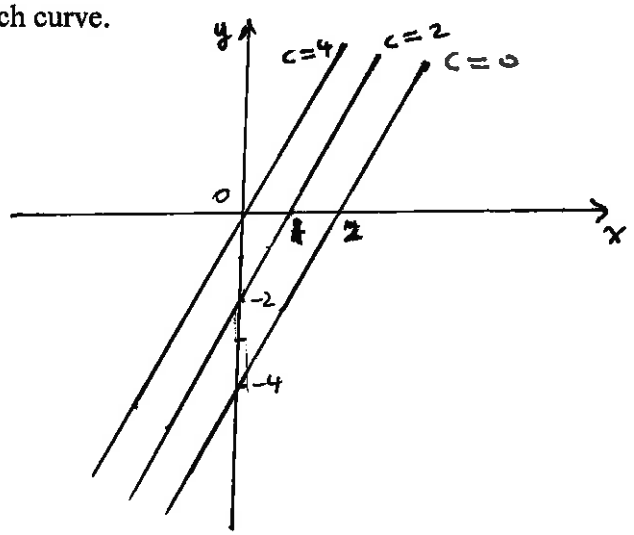
2. Let $f(x,y) = 4 - 2x + y$.

a. Determine the level sets for the function f for the values $c = 0, 2, 4$. Plot all of them in the same coordinate sytem, and make sure to indicate the height c of each curve.

$c = 0 \quad 0 = 4 - 2x + y \Rightarrow y = 2x - 4$

$c = 2 \quad 2 = 4 - 2x + y \Rightarrow y = 2x - 2$

$c = 4 \quad 4 = 4 - 2x + y \Rightarrow y = 2x$



b. Sketch the (explicit) graph of f on the back of the page.

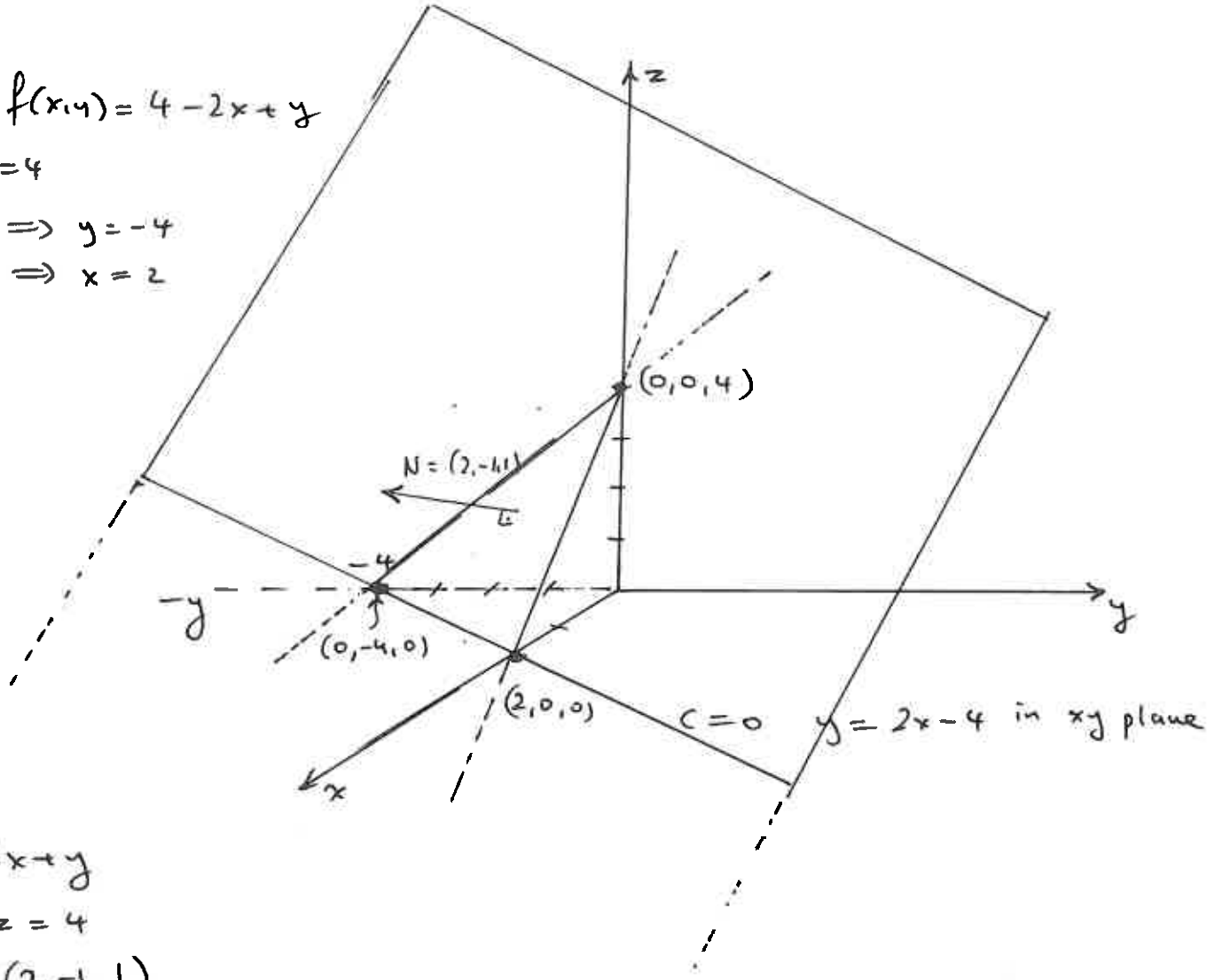
(2b)

$z = f(x,y) = 4 - 2x + y$

$f(0,0) = 4$

$x = 0, z = 0 \Rightarrow y = -4$

$y = 0, z = 0 \Rightarrow x = 2$

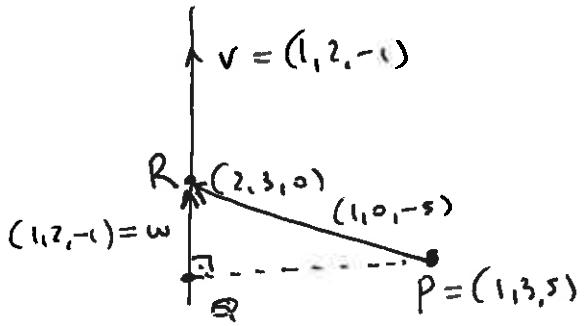


$z = 4 - 2x + y$

$2x - y + z = 4$

$N = (2,-1,1)$

(1b) METHOD II



$w = \text{proj}_v(R - P)$ ← $(2,3,0) - (1,3,5) = (1,0,-5)$

$= \text{proj}_{(1,2,-1)}(1,3,5)$

$= \dfrac{(1,0,-5)\cdot(1,2,-1)}{(1,2,-1)\cdot(1,2,-1)}(1,2,-1)$

$= \dfrac{1+0+5}{1+4+1}(1,2,-1) = (1,2,-1)$

$Q = R - w = (2,3,0) - (1,2,-1)$

$= (1,1,1)$

$\text{dist} = \|P - Q\| = \|(1,3,5) - (1,1,1)\| = \|(0,2,4)\| = \sqrt{20}$

$= 2\sqrt{5}.$

OR

$\text{dist} = \|(1,0,-5) - (1,2,-1)\| = \|(0,-2,-4)\| = 2\sqrt{5}.$