MATH 2850
Quiz 1
February 2, 2017

NAME. SOLUTIONS

Let $Q = (1,2,3)$ and $P = (0,-1,1)$ be given two points in $\mathbb{R}^3$.

1. a. Find a set of parametric equations for the line ℓ passing through Q and P.

$Q - P = (1,2,3) - (0,-1,1) = (1,3,2)$.

There are many solutions, here are some

$$\begin{cases} x = 1 + t \\ y = 2 + 3t \\ z = 3 + 2t \end{cases} \text{ OR } \begin{cases} x = 1 - t \\ y = 2 - 3t \\ z = 3 - 2t \end{cases} \text{ OR } \begin{cases} x = 0 + t \\ y = -1 + 3t \\ z = 1 + 2t \end{cases} \text{ OR } \begin{cases} x = 6 - 3t \\ y = 17 - 9t \\ z = 13 - 6t \end{cases} \text{ OR } \dots$$

b. Give a symmetric form for the line ℓ from part (**a**).

Again, there are many solutions:

$$\frac{x-1}{1} = \frac{y-2}{3} = \frac{z-3}{2} \text{ OR } \frac{x-0}{-1} = \frac{y+1}{-3} = \frac{z-1}{-2} \text{ OR } \dots$$

2. Do the lines $\ell_1 : \begin{cases} x = 3 + t \\ y = 2 + 2t \\ z = 5 + t \end{cases}$ and $\ell_2 : \begin{cases} x = 2 + t \\ y = -2 \\ z = 2 - t \end{cases}$ intersect? Explain why or why not.

If they intersect, find all points of intersection.

The question reduces in finding all solutions of $*$

$$* \begin{cases} 3 + t_1 = x = 2 + t_2 \\ 2 + 2t_1 = y = -2 \\ 5 + t_1 = z = 2 - t_2 \end{cases} \longrightarrow \begin{cases} 3 + t_1 = 2 + t_2 \\ 2 + 2t_1 = -2 \\ 5 + t_1 = 2 - t_2 \end{cases} \longrightarrow \begin{matrix} t_1 - t_2 = -1 & (1) \\ 2t_1 = -4 & (2) \\ t_1 + t_2 = -3 & (3) \end{matrix}$$

One can solve by using row-reduction (next page) (II) or a simpler approach (I)

(I) Simple approach

(2) $2t_1 = -4$

$t_1 = -2$

(1) becomes $t_1 - t_2 = -2 - t_2 = -1$

$\Rightarrow -1 = t_2$

(3) $t_1 + t_2 = (-1) + (-2) = -3$ is consistent with (1) $\times 2$

$$\longrightarrow \begin{matrix} x = 3 + t_1 = 3 - 2 = 1 \\ y = 2 + 2t_1 = 2 - 4 = -2 \\ z = 5 + t_1 = 5 - 2 = 3 \end{matrix}$$

$\downarrow$

$(1, -2, 3)$ is the only pt of intersection

II

$$t_1 - t_2 = -1$$
$$2t_1 = -4$$
$$t_1 + t_2 = -3$$

$$\left[\begin{array}{cc|c} 1 & -1 & -1 \\ 2 & 0 & -4 \\ 1 & 1 & -3 \end{array}\right] \xrightarrow[R_1 \leftrightarrow R_2]{} \left[\begin{array}{cc|c} 2 & 0 & -4 \\ 1 & -1 & -1 \\ 1 & 1 & -3 \end{array}\right] \xrightarrow[\frac{1}{2}R_1]{} \left[\begin{array}{cc|c} 1 & 0 & -2 \\ 1 & -1 & -1 \\ 1 & 1 & -3 \end{array}\right]$$

$$\xrightarrow[\substack{R_2 - R_1 \\ R_3 - R_1}]{} \left[\begin{array}{cc|c} 1 & 0 & -2 \\ 0 & -1 & 1 \\ 0 & 1 & -1 \end{array}\right] \xrightarrow[R_3 + R_2 \to R_3]{} \left[\begin{array}{cc|c} 1 & 0 & -2 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{array}\right] \xrightarrow[-R_2]{} \left[\begin{array}{cc|c} 1 & 0 & -2 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{array}\right]$$

$$t_1 = -2$$
$$t_2 = -1$$

$$x = 3 + t_1 = 2 + t_2 = 1$$
$$y = 2 + t_1 = -2 \quad = -2$$
$$z = 5 + t_1 = 2 - t_2 = 3$$

The only pt of intersection is $(1, -2, 3)$.