MATH 2850

Some Practice questions for the Final Exam

Problem 1. Sketch the domain of integration of the following integral and express it in spherical coordinates as an iterated integral with bounds. Compute the value of this integral.

$$\int_0^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} z(x^2+y^2)\,dz\,dy\,dx$$

Problem 2. a. Give a parametrization of the line segment C from the $(1,0,0)$ to $(1,3,4)$.

b. Calculate the line integral $\int_C \mathbf{F}\cdot d\mathbf{s}$ along the curve C of part (**a**), where $\mathbf{F}(x,y,z) = (-y,-z,x)$.

Problem 3. a. Give a parametrization of the cylinder S described by $x^2+y^2 = 4$ and $0 \le z \le 3$. Sketch the surface and specify the parametrization domain.

b. Find a closed equation ($Ax+By+Cz = D$) for the tangent plane to S at the point $(\sqrt{3},1,2)$.

Problem 4.a. Calculate the divergence of $\mathbf{F}(x,y,z) = xyz\mathbf{i} + x\cos y\mathbf{j} + e^{2z}\mathbf{k}$.

b. Prove that $\nabla\times\nabla f = 0$, for every twice continuously differentiable function $f:\mathbf{R}^3\to\mathbf{R}$.

Problem 5. Let B be the subset of $\mathbf{R}^3$, defined by $x+y\le 2$, $z\le 4-x^2$ and $x\ge 0$, $y\ge 0$, $z\ge 0$.

a. Sketch the region B. **b.** Calculate the volume of B.

Problem 6. a. Is $\mathbf{F}(x,y,z) = (z+yz, xz, x+xy)$ a conservative vector field? If it is, find a potential function for $\mathbf{F}$. If it is not, explain why.

b. Calculate the line integral $\int_C \mathbf{F}\cdot d\mathbf{s}$, where C is the curve given by $g(t) = (\cos^3 t, \sin^4 t, \ln(1+t)) \;:\; [0,\pi]\to\mathbf{R}^3$.

Problem 7. Let $g(u,v) = (u,v,v^2-u^2)$ parametrize a surface S for $0\le u\le 2$ and $0\le v\le 2$.

a. Calculate $\int_S \sqrt{1+4z+8x^2}\;dS$.

b. Calculate the *flux* through the surface S, $\iint_S \mathbf{F}\cdot d\mathbf{S}$, where $\mathbf{F}(x,y,z) = (2x,z,y)$.

Problem 8. Verify that the Stokes' Theorem holds (by computing both integrals and showing that they are equal) for the vector field $\mathbf{F}(x,y,z) = (x,y,0)$ and for the surface S parametrized by $g(u,v) = (u,v,u^2+v^2)$ for $u^2+v^2\le 4$. Sketch S and its border.

Problem 9. Find $\int_{C_i} (x^2-y^3)dx + (x^3-y^2)dy$, for each of the following curves.

a. C_1 is the unit circle $x^2+y^2 = 1$, traced counterclockwise.

b. C_2 is the part of the unit circle $x^2+y^2 = 1$ in the upper half-plane ($y\ge 0$) traced clockwise from $(-1,0)$ to $(1,0)$.

Problem 10. Calculate $\iint_D (2x+y)\cos(y-x)dA$ where D is the parallelogram with vertices $(0,0),(2,2),(3,0)$ and $(1,-2)$. (Hint: Find the equations of the edges, and make a 2 variable substitution.)

Problem 11. Verify Gauss' Theorem for the 3-dimensional region $D = \{(x,y,z) : x^2+y^2 = 1 \leq z \leq 5\}$ and vector field $\mathbf{F}(x,y,z) = x^2\mathbf{i} + y\mathbf{j} + z\mathbf{k}$.

Problem 12. Let $\mathbf{G} = (2xy - e^x, x^2 + 4y)$.

a. Is $\mathbf{G}$ a conservative vector field? Explain why, if it is not. Find a potential function for it, if it is conservative.

b. Calculate $\int_C \mathbf{G} \cdot \mathbf{ds}$ where C is the curve from $(0,0)$ along x-axis to the point $(3,0)$ and then following the circle $x^2+y^2 = 9$ counterclockwise to the point $(0,-3)$.

Problem 13. Compute $\oint_C x^2dx + x^2ydy$, where the curve C traces the boundary of the triangle with vertices $(0,0)$, $(2,2)$ and $(0,2)$, counterclockwise.

Problem 14. Convert the following integral to polar coordinates, and compute it.

$$\int_0^2 \int_0^{\sqrt{4-x^2}} (2x^2+y^2)dydx$$

Problem 15.

a. Give a parametrization for the surface $z^2 = x^2+y^2$ where $0 \leq z \leq 3$.

b. Set up the area integral $\iint_S 1dS$ as an iterated integral with bounds, according to the parametrization you obtained, and evaluate this integral.

Problem 16. Compute the flux integral $\iint_S \nabla \times \mathbf{F} \cdot d\mathbf{S}$, where $\mathbf{F}(x,y,z) = (x^2, z^2, y^2)$, and S is the hemisphere $z = \sqrt{1-x^2-y^2}$. Assume that S is oriented with normal pointing away from the origin.

Problem 17. For the curve L given by $g(t) = (t, t^2, \frac{2}{3}t^3)$, $0 \leq t \leq 1$, calculate the following.

a. $\int_L (x^2+y)\, ds$

b. $\int_L e^x dx + zdy + \sin z dz$

Problem 18. Let P be the part of the graph $z = x^2+y$ that lies above the square $1 \leq x \leq 2$ and $0 \leq y \leq 1$ in the xy-plane, with normal pointing upwards.

a. Give a parametrization of P with the specified normal.

b. Find the flux of the vector field $\mathbf{F}(x,y,z) = -x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$, through P.

Problem 19. For the given integral $\int_0^2 \int_{y/2}^1 e^{-x^2}\, dx\, dy$:

a. Sketch the domain of the integral.
b. Reverse the order of integration.
c. Find the value of this integral.

Problem 20. Sketch the region D defined by $x^2 + y^2 \leq 4$, $x \geq 0$ and $y \geq 0$ in $\mathbf{R}^2$, describe the region D by using polar coordinates, and calculate $\iint_D x^2 \sqrt{x^2 + y^2}\, dx\, dy$.

Problem 21. Let B be the region inside the sphere $x^2 + y^2 + z^2 = 8$ and above the cone $z = \sqrt{x^2 + y^2}$ in $\mathbf{R}^3$. Set up iterated integrals for the triple integral $\int_B x^2 + y^2\, dV$ in the following coordinates.

DO NOT EVALUATE THE INTEGRALS.

a. Rectangular (x,y,z) coordinates
b. Cylindrical coordinates
c. Spherical coordinates

Problem 22. Let a transformation T from the uv-plane to the xy-plane be defined by $x = u + v$ and $y = u - v$. Let R_{uv} be the rectangular region given by $0 \leq u \leq 2$, and $0 \leq v \leq 1$ in the uv-plane.

a. Find and sketch the region $R_{xy} = T(R_{uv})$, the image of R_{uv} under the transformation T.
b. Find $\frac{\partial(x,y)}{\partial(u,v)}$.
c. Transform $\int_{R_{xy}} xy\, dx\, dy$ to an integral over R_{uv}. Write the integral as an iterated integral, but do not evaluate it.

Problem 23. For the curve L parametrized by $g(t) = (2t, \ln t, t^2)$, for $1 \leq t \leq e$, calculate the following.

a. The length of L.
b. The mass of a wire along the curve L with density z at a point (x,y,z).
c. The line integral $\int_L x\, dx + (x^2 + z)dy$

Problem 24. a. Is the vector field $\mathbf{F}(x,y) = (y\cos xy,\ 2 + x\cos xy) : \mathbf{R}^2 \to \mathbf{R}^2$ conservative? Explain why if it is not. Find a potential function for it, if it is conservative.

b. By using part (a), calculate the line integral $\int_C (y\cos xy)dx + (2 + x\cos xy)dy$, where C is the curve given by $g(t) = (\pi(1-t)^3,\ e^{2t}) : [0,1] \to \mathbf{R}^3$.

Problem 25. Find $\int_C (e^{x^2} - y)dx + (e^{y^2} + 2x)dy$, where C is boundary of the square with vertices $(0,0)$, $(3,0)$, $(3,3)$ and $(0,3)$ in the xy-plane traversed counterclockwise.

MATH 2850

Final

Fall 2016

Problem 1. Find an equation (of the form $Ax + By + Cz = D$) for the plane tangent to the surface

$$x = e^s$$
$$y = t^2 e^{2s}$$
$$z = e^{-s} + t$$

at the point $(1, 1, 0)$.

Problem 2. Find the flux $\iint_S \mathbf{F} \cdot d\mathbf{S}$ of $\mathbf{F}(x, y, z) = 2x\,\mathbf{i} + y\,\mathbf{j} + z\,\mathbf{k}$ across the surface S consisting of the triangular region of the plane $2x + 2y + z = 2$ that is cut out by the coordinate planes. Use the upward-pointing normal to orient S.

Problem 3. Verify the Stokes's Theorem (by computing both integrals and showing that they are equal) for the surface S defined by $x^2 + y^2 + z = 1$, $z \geq 0$, oriented by upward normal, and the vector field $\mathbf{F}(x, y, z) = xz\,\mathbf{i} + yz\,\mathbf{j} + (x^2 + y^2)\,\mathbf{k}$.

Problem 4.

a. Determine whether $\mathbf{F}(x, y, z) = (2x + y)\,\mathbf{i} + (x + z\cos yz)\,\mathbf{j} + (y\cos yz)\,\mathbf{k}$ is a conservative vector field. If it is, find a scalar potential function for $\mathbf{F}$. If it is not, explain why.

b. Calculate the line integral $\int_C \mathbf{F} \cdot ds$, where C is the curve given by $\mathbf{x}(t) = (t, \cos t, \sin t) : [0, \frac{\pi}{2}] \to \mathbf{R}^3$.

Problem 5. Calculate $\oint_C (\sin(x^2) + e^y)dx + (x^2y - e^{y^2})dy$, where C is the path formed by following the edges of the rectangle with vertices $(0, 0)$, $(1, 0)$, $(1, 2)$, and $(0, 2)$ oriented clockwise.

Problem 6. a. Give a parametrization of the curve C obtained by intersecting the plane $z = 2y$ and the cylinder $x^2 + y^2 = 4$ and oriented counterclockwise around the $z-$axis as seen from the positive $z-$axis. Sketch the curve C.

b. Calculate the line integral $\int_C \mathbf{F} \cdot \mathbf{ds}$ along the curve C of part (**a**), where $\mathbf{F}(x, y, z) = 3z\,\mathbf{i} + 2x\,\mathbf{j} + y\,\mathbf{k}$.

Problem 7. Evaluate the integral $\int_0^1 \int_{x/2}^{(x/2)+1} x^4(2y - x)e^{(2y-x)^2}\,dydx$ by making the substitution $u = 2y - x$ and $v = x$.

Problem 8. Let $\iiint_W (x^2 + y^2 + z^2) dV$ be given where W is the region inside the sphere $x^2 + y^2 + z^2 = 1$ and above the cone $z = \sqrt{x^2 + y^2}$.

a. Sketch the domain of integration W.

b. Express this integral in spherical coordinates as an iterated integral with bounds.

c. Express this integral in cylindrical coordinates as an iterated integral with bounds.

d. Compute the value of this integral.

Problem 9.

a. Evaluate $\int_{-a}^{a} \int_{-\sqrt{a^2-x^2}}^{\sqrt{a^2-x^2}} 4y^2 \, dy \, dx$

b. Evaluate $\int_{0}^{1} \int_{y}^{1} x^2 \sin xy \, dx \, dy$

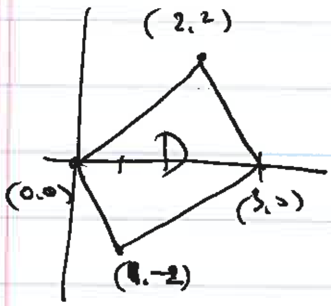
Problem 10.

Let $\mathbf{F}(x, y, z) = x\,\mathbf{i} + y\,\mathbf{j} + (3 - 2z)\,\mathbf{k}$, and let S be defined by $z = e^{1-x^2-y^2}$, $z \geq 2$, oriented upward normal. Calculate $\iint_S \mathbf{F} \cdot d\mathbf{S}$.(HINT: What is the divergence of F?)

Review
5/9/17

①

Practice #10 $\iint (2x+y)\cos(y-x)$



$u = 2x+y$

$v = y-x$

$\underbrace{\frac{\partial(u,v)}{\partial(x,y)} = \begin{vmatrix} 2 & 1 \\ -1 & 1 \end{vmatrix} = 3}$

$du\,dv = 3\,dx\,dy$

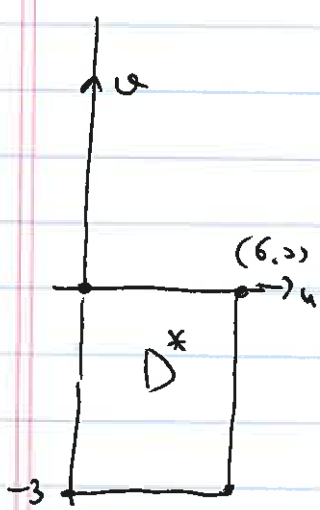
$T(x,y) = (2x+y, y-x)$

$T(0,0) = (0,0)$

$T(1,-2) = (0,-3)$

$T(2,2) = (6,0)$

$T(3,0) = (6,-3)$



$\left.\begin{array}{l} T \text{ linear} \\ \text{and Edges of } D \\ \text{are lines} \end{array}\right\} \Rightarrow$ edges of D^* are line segments

$$\iint (2x+y)\cos(y-x)\,dx\,dy = \int_{-3}^{0}\int_0^6 u \cos v \, du\, dv \, \frac{1}{3}$$

$$= \frac{1}{3}\left(\int_0^6 u\,du\right)\left(\int_{-3}^{0} \cos v\,dv\right)$$

$$= \frac{1}{3}\left(\frac{u^2}{2}\Big|_0^6\right)\left(\sin v\Big|_{-3}^{0}\right) = \frac{1}{3}\cdot 18 \cdot (0 - \sin(-3))$$

$$= 6\sin 3$$

Practice # 2

$(1,0,0) \longrightarrow (1,3,4)$

$(1,3,4)-(1,0,0) = (0,3,4)$

$\vec{x} = (1,0,0) + t(0,3,4) \quad 0 \le t \le 1$

$\vec{x} = (\underbrace{1, 3t, 4t})$ $\quad F = (-y, -z, x)$

$$\int \vec{F} \cdot d\vec{s} = \int F(\vec{x}(t)) \cdot \vec{x}'(t)\, dt$$

$F(\vec{x}(t)) = F(1, 3t, 4t) = (-3t, -4t, 1)$

$x'(t) = (0,3,4)$

$$\int_0^1 (-3t, -4t, 1) \cdot (0,3,4)\, dt = \int_0^1 -12t + 4 \; dt$$

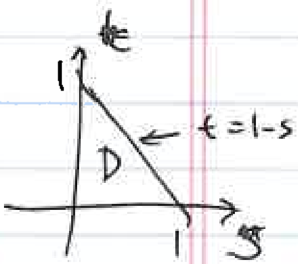
$$= -\frac{12t^2}{2} + 4t \Big|_0^1$$

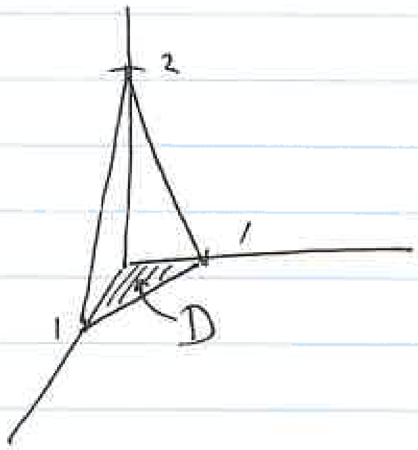
$$= -6 + 4 = -2.$$

Final 2016 #2

$$F = 2x\, i + y\, j + z\, k$$

$$S \quad 2x + 2y + z = 2$$





$$z = 2 - 2x - 2y$$

$$\Phi = (s, t, 2 - 2s - 2t)$$

$$\overset{\Vert}{x} \quad \overset{\Vert}{t}$$

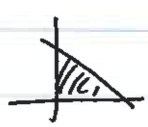
$$\Phi_s = (1, 0, -2)$$

$$\Phi_t = (0, 1, -2)$$

$$N = (2, 2, 1)$$

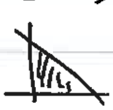
$$\iint \vec{F} \cdot d\vec{S} = \iint F(\Phi(s,t)) \cdot N_{\Phi}(s,t)\, ds\, dt$$

$$= \iint (2s, t, 2 - 2s - 2t) \cdot (2, 2, 1)\, ds\, dt$$



$$= \iint (4s + \cancel{2t} + 2 - 2s - \cancel{2t})\, ds\, dt$$

$$= \iint (2s + 2)\, ds\, dt = \int_0^1 \int_0^{1-s} (2s+2)\, dt\, ds$$



$$= \int_0^1 (1-s)(2s+2)\, ds = \int_0^1 2(1 - s^2)\, ds$$

$$= 2\left(s - \frac{s^3}{3}\Big|_0^1\right) = \frac{4}{3}.$$

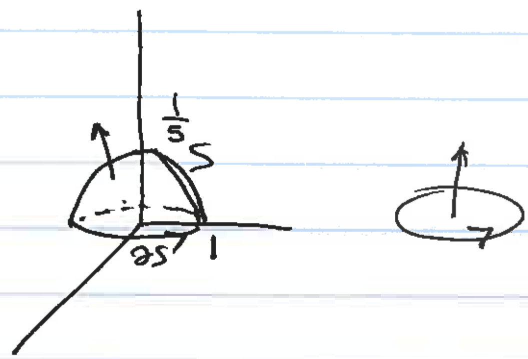
p 505 7.3 Exc #1

$S: \quad x^2+y^2+5z=1 \quad z \geq 0 \quad \uparrow N$

$\vec{F} = xz\,i + yz\,j + (x^2+y^2)\,\vec{k}$

Verify: $\displaystyle\int_{\partial S} \vec{F}\cdot d\vec{s} = \iint_S (\nabla\times F)\,d\vec{S}$ STOKES'

$z = \dfrac{1-(x^2+y^2)}{5}$



$\partial S: \quad (\overset{x}{\cos t}, \overset{y}{\sin t}, \overset{z}{0}) = \vec{x}(t)$

$(-\sin t, \cos t, 0) = x'(t)$

$F(\vec{x}(t)) = (0, 0, 1)$

$\displaystyle\int_{\partial S} \vec{F}\cdot d\vec{s} = \int_0^1 (0,0,1)\cdot(-\sin t, \cos t, 0)\,dt = 0$

$\nabla\times F = \begin{vmatrix} i & j & k \\ \partial_x & \partial_y & \partial_z \\ xz & yz & x^2+y^2 \end{vmatrix}$

$= (2y - y, x - 2x, 0) = (y, -x, 0)$

$$\underline{X}(s,t) = \left(s,\ t,\ \frac{1-(s^2+t^2)}{5} \right) \qquad s^2+t^2 \le 1$$

$$\underline{X}_s = \left(1, 0, \frac{-2s}{5}\right)$$

$$\underline{X}_t = \left(0, 1, -\frac{2t}{5}\right)$$

$$N = \left(\frac{2s}{5}, \frac{2t}{5}, 1\right)$$

$$\iint (\nabla \times F)\cdot d\vec{S} = \iint_{s^2+t^2\le 1} \underbrace{(t,-s,0)\cdot\left(\frac{2s}{5},\frac{2t}{5},1\right)}_{0} dt\,ds = 0.$$

$$\nabla \times F \longrightarrow (y, -x, 0)$$

$$(\nabla \times F)(\underline{X}(s,t)) = (t, -s, 0)$$

remark

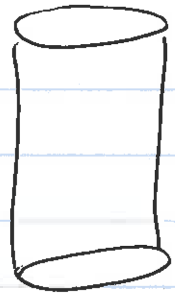
$$\boxed{\begin{aligned} &\int_{-1}^{1}\int_{-\sqrt{1-s^2}}^{\sqrt{1-s^2}} f(s,t)\,dt\,ds \qquad \text{if } f(s,t)\not\equiv 0 \\ &\int_0^{2\pi}\int_0^1 f(r\cos\theta, r\sin\theta)\, r\,dr\,d\theta \end{aligned}}$$

verified

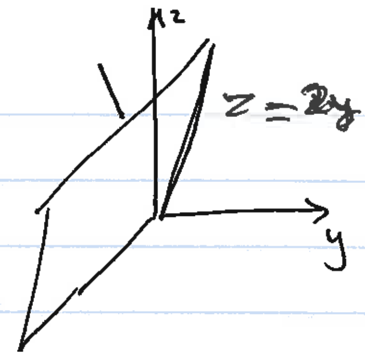
$$0 = \iint_S \vec{\nabla\times F}\cdot\vec{dS} = \oint_{\partial S} \vec{F}\cdot\vec{ds} = 0$$

2016 Final

6a)



$x^2+y^2=4$



$$x(t)=(\underbrace{2\cos t,\ \overbrace{2\sin t}^{y}}_{\text{top view}},\ \overbrace{4\sin t}^{z=2y}) \qquad 0\le t\le 2\pi$$

$$F=(3z,2x,y)$$

6b)

$$\int_C F\cdot ds=\int_0^{2\pi}\underbrace{(12\sin t,\ 4\cos t,\ 2\sin t)}_{F(\vec{x}(t))}\cdot(-2\sin t,\ 2\cos t,\ 4\cos t)$$

$$x'(t)=(-2\sin t,\ 2\cos t,\ 4\cos t)$$

$$=\int_0^{2\pi}-24\sin^2 t+8\cos^2 t+8\cos t\sin t\,dt$$

$$=\int_0^{2\pi}-24\cdot\frac{1-\cos 2t}{2}+8\cdot\frac{1+\cos 2t}{2}+8\sin t\cos t\,dt$$

$\downarrow\int$ $u=\sin t$

$4\sin^2 t$

$$=(-12+\cancel{8}4)\cdot 2\pi+0$$

$$=-16\pi$$

2016 Final # 6

Method B via STOKES' THM

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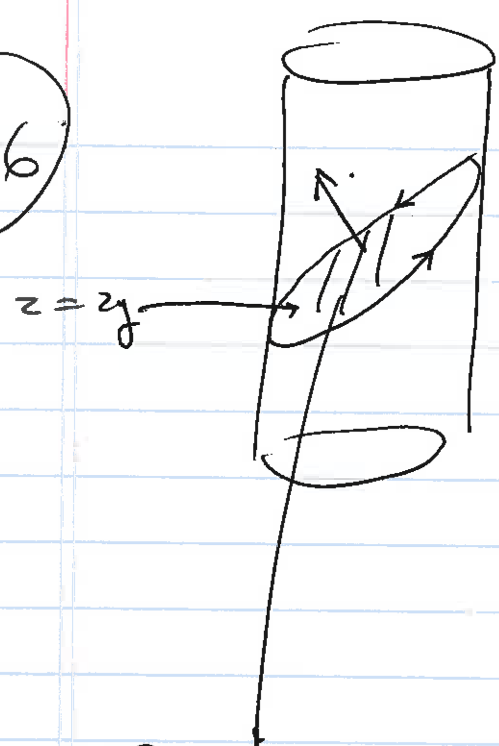


$$F = (3z, 2x, y)$$

$$\operatorname{curl} F = \begin{vmatrix} i & j & k \\ \partial_x & \partial_y & \partial_z \\ 3z & 2x & y \end{vmatrix}$$

$$= (1, 3, 2)$$

$$X = (s, t, 2t) \qquad s^2 + t^2 \leq 4$$

$$X_s = (1, 0, 0)$$

$$X_t = (0, 1, 2)$$

$$N = (0, -2, 1)$$

$$\iint \vec{\nabla} \times F \cdot d\vec{S} = \iint \underbrace{(1, 3, 2) \cdot (0, -2, 1)}_{-6+2} \, dA$$

$$= \iint -4 \, dA = -4 \cdot \text{area } D$$

$$= -4 \cdot 4\pi = -16\pi$$

6.2

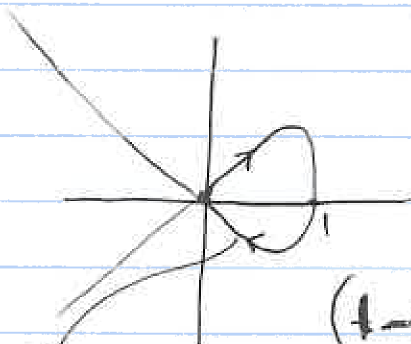
#15

$$((1-t^2), t^3-t) = \vec{x}(t)$$

$-1 \le t \le 1$

$$1-t_1^2 = 1-t_2^2$$
$$t_1^2 = t_2^2$$
$$t_1 = \pm t_2$$

$$\vec{x}(-1) = (0,0) = \vec{x}(1)$$



$$(1-t^2, -t(1-t^2))$$

clockwise

$$-\int P\,dx + Q\,dy = \iint \underbrace{\left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right)}_{1} dx\,dy = \text{Area}$$

$$\frac{\partial Q}{\partial x} = 1 \qquad Q = x$$

$$\frac{\partial P}{\partial y} = 0 \qquad P = 0$$

$$\text{Area} = -\int_{-1}^{1} x\,dy = -\int_{-1}^{1} (1-t^2)(3t^2-1)\,dt$$

$$x = 1-t^2$$
$$y = t^3 - t \qquad dy = (3t^2-1)\,dt$$

$$= -\int_{-1}^{1} 3t^2 - 1 - 3t^4 + t^2\,dt = -\int_{-1}^{1} 4t^2 - 1 - 3t^4\,dt$$

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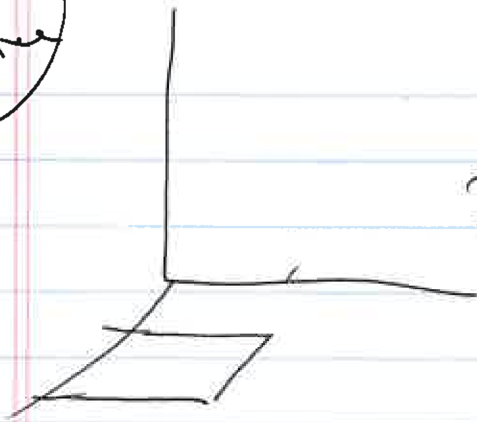
$$-\left(\frac{4t^3}{3} - t - \frac{3}{5}t^5 \Big|_{-1}^{1}\right) = 2\left(\frac{4}{3} - 1 - \frac{3}{5}\right)$$

$$= -2\left(\frac{1}{3} - \frac{3}{5}\right)$$

$$= 2 \cdot \frac{+4}{15} = \frac{+8}{15} = \text{area}$$

#18 practice



$$\underline{X} = (s, t, s^2+t) \qquad 1 \le s \le 2, \quad 0 \le t \le 1$$

$$\underline{X}_s = (1, 0, 2s)$$
$$\underline{X}_t = (0, 1, 1)$$

$$N_{\underline{X}} = (-2s, -1, 1) \quad \text{normal } \uparrow.$$

$$Y = (t, s, t^2+s) \qquad N \downarrow.$$
$$Y_s = (0, 1, 1)$$
$$Y_t = (1, 0, 2t)$$

$$N = (2t, 1, -1) \quad N\downarrow.$$

use $\underline{X}$

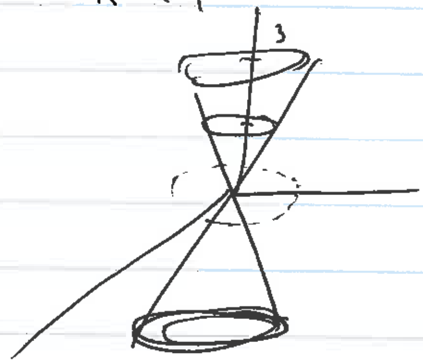
$$\int_{\underline{X}} \vec{F} \cdot \overline{dS} = \int_1^2 \int_0^1 (-s, t, s^2+t) \cdot (-2s, -1, 1)\, dt\, ds$$

$$F = (-x, y, z)$$
$$F(\underline{X}(s,t)) = (-s, t, s^2+t)$$

$$= \int_1^2 \int_0^1 (+2s^2 \cancel{-t} + s^2 \cancel{+t})\, dt\, ds$$

$$= \int_1^2 \int_0^1 3s^2\, dt\, ds = \left(\int_1^2 3s^2\, ds\right)\Big(\underbrace{\int_0^1 dt}_{1}\Big) = s^3 \Big|_1^2 = 7$$

Practice #15 $\quad z^2 = x^2 + y^2$



Two easy parametrizations

a) $\underline{X} = (\underbrace{s\cos t,\ s\sin t}_{\text{circle of radius } s.},\ s)$ $\quad$ at height s $\qquad 0 \le s \le 3$, $\ 0 \le t \le 2\pi$

OR

b) $\underline{Y} = (s, t, \sqrt{s^2+t^2})$ $\qquad s^2 + t^2 \le 9$

I

$$\underline{X}_s = (\cos t, \sin t, 1)$$
$$\underline{X}_t = (-s\sin t, s\cos t, 0)$$
$$N = (-s\cos t, -s\sin t, s) \qquad \|N\| = \sqrt{s^2\cos^2 t + s^2\sin^2 t + s^2} = \sqrt{2s^2} = s\sqrt{2}$$

$$\text{Area} = \int_0^{2\pi}\int_0^3 \|N\|\, ds\, dt$$

$$= \left(\int_0^{2\pi} dt\right)\left(\int_0^3 s\sqrt{2}\, ds\right)$$

$$= 2\pi \cdot \frac{s^2}{2}\Big|_0^3 \sqrt{2} = 9\pi\sqrt{2}$$

OR

PTO for $\underline{Y}$

b)

$$Y = (s, t, \sqrt{s^2+t^2})$$

$$Y_s = \left(1, 0, \frac{2s}{2\sqrt{s^2+t^2}}\right)$$

$$Y_t = \left(0, 1, \frac{2t}{2\sqrt{s^2+t^2}}\right)$$

$$N = \left(\frac{s}{\sqrt{s^2+t^2}}, \frac{t}{\sqrt{s^2+t^2}}, 1\right)$$

$$\|N\| = \sqrt{\frac{s^2}{s^2+t^2} + \frac{t^2}{s^2+t^2} + 1} = \sqrt{2}$$

$$A = \iint_{s^2+t^2 \le 9} \sqrt{2} = \sqrt{2} \text{ area } \pi \cdot 3^2 = 9\pi\sqrt{2}$$

Practice #17

$$g(t) = (\overset{x}{\overset{\parallel}{t}}, \overset{y}{\overset{\parallel}{t^2}}, \tfrac{2}{3}t^3) \qquad 0 \le t \le 1$$

(a)

$$g'(t) = (1, 2t, 2t^2)$$

$$|g'(t)| = \sqrt{1 + 4t^2 + 4t^4} = 1 + 2t^2$$

$$\int_L (x^2+y)\, ds = \int_0^1 \underbrace{(t^2+t^2)}_{(x^2+y)\circ g(t)} \underbrace{(1+2t^2)\, dt}_{ds}$$

$$= \int_0^1 2t^2 + 4t^4 \, dt$$

$$= \frac{2}{3} + \frac{4}{5} = \frac{22}{15}$$

(b)

$$\int e^x dx + z\,dy + \sin z\, dz$$

$x = t \qquad dx = dt$

$y = t^2 \qquad dy = 2t\,dt$

$z = \frac{2}{3}t^3 \qquad dz = 2t^2 dt$

$$= \int_0^1 \underbrace{e^t}_{e^x} \underbrace{dt}_{dx} + \underbrace{\tfrac{2}{3}t^3}_{z} \cdot \underbrace{2t\,dt}_{dy} + \underbrace{\underbrace{(\sin \tfrac{2}{3}t^3)}_{\sin z} \cdot \underbrace{2t^2 dt}_{dz}}_{u = \frac{2}{3}t^3}$$

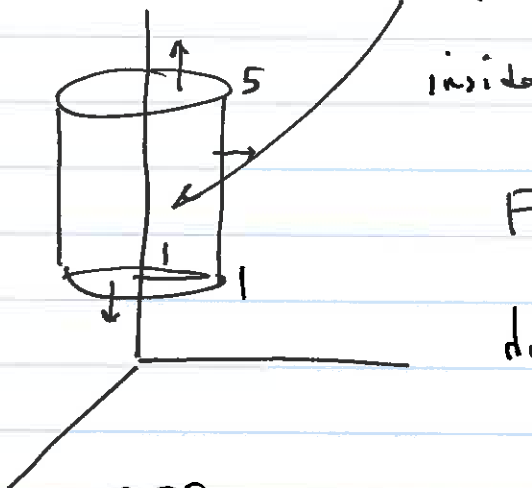
$$= e^t + \frac{4}{3}\frac{t^5}{5} - \cos \frac{2}{3}t^3 \Big|_0^1$$

$$= \left(e + \frac{4}{15} - \cos\frac{2}{3}\right) - (\cancel{1} + 0 - \cancel{1})$$

Practice #11

Verify Gauss' Thm

$$D = \left\{ (x,y,z) \;\middle|\; \begin{array}{l} x^2+y^2 \le 1 \\ 1 \le z \le 5 \end{array} \right\}$$



$F = (x^2, y, z)$.

$\operatorname{div} F = 2x + 2$

<u>Check</u> $\displaystyle\iiint_D \operatorname{div} F \cdot dV = \iint_{S=\partial D} F \cdot dS$

$$\iiint_D (2x+z)\, dV = \int_0^{2\pi}\int_0^1\int_1^5 (2\underbrace{r\cos\theta}_{x}+z)\, r \cdot dz\, dr\, d\theta$$

2 summands A, B

(A) $\displaystyle\int_0^{2\pi}\int_0^1\int_1^5 2r^2\cos\theta \, dz\, dr\, d\theta$

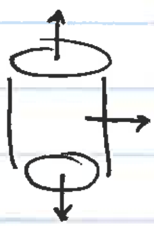
$$= \int_0^1\int_1^5 \underbrace{\int_0^{2\pi} 2r^2\cos\theta\, d\theta}_{0}\, dz\, dr = 0$$

$$B = \int_0^{2\pi}\int_0^1\int_1^5 2r\,dz\,dr\,d\theta$$

$$= \left(\int_0^{2\pi} d\theta\right)\left(\int_0^1 2r\,dr\right)\left(\int_1^5 dz\right)$$

$$= 2\pi \cdot 1 \cdot 4 = 8\pi$$

$$A + B = 8\pi = \iiint \operatorname{div} F \cdot dV$$



$$\iint_{\partial D = S} F \cdot dS$$

$X = (s, t, 5)$	$Y = (s, t, 1)$	$Z = (\cos s, \sin s, t)$
$X_s = (1, 0, 0)$	$Y_s = (1, 0, 0)$	$Z_s = (-\sin s, \cos s, 0)$
$X_t = (0, 1, 0)$	$Y_t = (0, 1, 0)$	$Z_t = (0, 0, 1)$
$N_X = (0, 0, 1) \uparrow$	$N = (0, 0, 1) \uparrow$	$N_Z = (\cos s, \sin s, 0)$
correct N	wrong normal	outward / correct

$$\iint_S F \cdot dS = \iint_X - \iint_Y + \iint_Z$$

$$\underbrace{5\pi - \pi + 4\pi}_{\text{see next pages}} = 8\pi$$

$$F = (x^2, y, z)$$

$$\iint_X \vec{F} \cdot d\vec{S} = \iint_{s^2+t^2 \le 1} (s^2, t, 5) \cdot (0,0,1) \, ds\,dt$$

$$= \iint_{s^2+t^2 \le 1} 5 \, ds\,dt = 5 \text{ area} = 5\pi$$

$$\iint_Y F \cdot dS = \iint_{s^2+t^2 \le 1} (s^2, t, 1) \cdot (0,0,1) = \pi$$

$$\iint_Z F \cdot dS = \iint_{\substack{0 \le s \le 2\pi \\ 1 \le t \le 5}} (\cos^2 s, \sin s, t) \cdot (\cos s, \sin s, 0)$$

$$(\underbrace{\cos s}_{x}, \underbrace{\sin s}_{y}, \underbrace{t}_{z})$$

$$= \int_1^5 \int_0^{2\pi} (\cos^3 s + \sin^2 s) \, ds\,dt$$

$$\int_0^{2\pi} \cos^3 s \, ds = \int_0^{2\pi} (1 - \sin^2 s) \cos s \, ds = \int_0^0 (1-u^2)\, du = 0.$$

$$u = \sin s$$

$$\int_0^{2\pi} \sin^2 s = \int_0^{2\pi} \frac{1 - \cos 2s}{2} \, ds = \pi$$

$$\int_1^5 \pi \, dt = 4\pi.$$