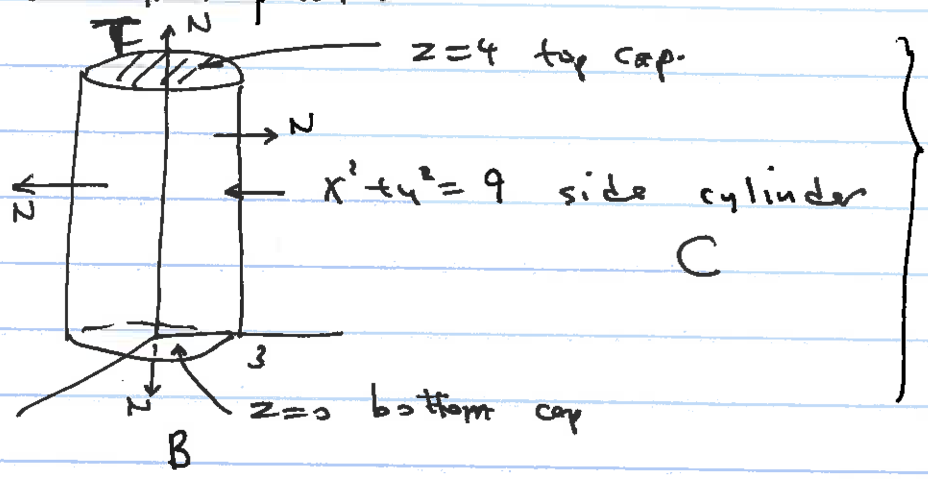
Ex 7.2 #14 p489.

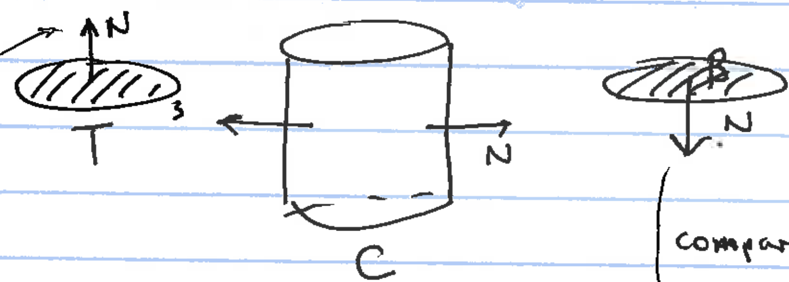
Want Outward normal



closed cylinder S

$$\iint (x\vec{i}+y\vec{j})\cdot d\vec{S}$$

$S = T \cup C \cup B$



compare

T: $X(s,t) = (s, t, 4)$

$X_s = (1,0,0)$

$X_t = (0,1,0)$

$N_{X} = (0,0,1)$

correct normal

Compare

use $+\iint_X \vec{F}\cdot d\vec{S}$

B: $Y(s,t) = (s, t, 0)$

$Y_s = (1,0,0)$

$Y_t = (0,1,0)$

$N_Y = (0,0,1)$ upwards

opposite normal

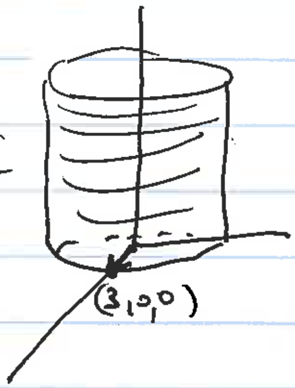
use $-\iint_Y \vec{F}\cdot d\vec{S}$

Side cylinder

$$C: \quad \Sigma = (3\cos t,\ 3\sin t,\ s) \qquad \left.\begin{array}{l} 0 \le t \le 2\pi \\ 0 \le s \le 4 \end{array}\right\}$$

$$\Sigma_s = (0, 0, 1)$$
$$\Sigma_t = (-3\sin t,\ +3\cos t,\ 0)$$
$$N = \Sigma_s \times \Sigma_t = (-3\cos t,\ -3\sin t,\ 0)$$

C



Take a test pt:

$$\Sigma(0,0) = (3, 0, 0)$$
$$N(0,0) = (-3, 0, 0)$$

Normal inwards

<u>opposite of what we want</u>

use

$$\iint_S \vec{F}\cdot d\vec{S} = + \iint_X F\cdot dS - \iint_Y F\cdot dS - \iint_Z F\cdot dS$$

$$F(x,y,z) = (x,y,0)$$

$$\iint_X F\cdot dS = \iint_{s^2+t^2\le 9} F(\Sigma(s,t))\cdot N_{\Sigma}(s,t)\,ds\,dt$$

$$= \iint_{s^2+t^2\le 9} \underbrace{(s,t,0)\cdot(0,0,1)}_{0}\,ds\,dt = \iint_{s^2+t^2\le 9} 0\,ds\,dt = 0$$

$\boxed{F(x,y,z) = (x,y,0)}$ $\boxed{Y(s,t) = (s,t,0)}$

$$\iint_Y F\cdot dS = \iint_{s^2+t^2\le 4} \underbrace{\overbrace{(s,t,0)}^{F(Y(s,t))}\cdot\overbrace{(0,0,1)}^{N_Y}}_{0} = 0.$$

$\boxed{Z(s,t) = (3\cos t, 3\sin t, s)}$

$$\iint_Z F\cdot dS = \iint_{\substack{0\le t\le 2\pi\\ 0\le s\le 4}} F(Z(s,t))\cdot N_Z(s,t)\, ds\,dt$$

$$= \iint_{\substack{0\le t\le 2\pi\\ 0\le s\le 4}} (3\cos t, 3\sin t, 0)\cdot(-3\cos t, -3\sin t, 0)\, ds\,dt$$

$$= \int_0^{2\pi}\int_0^4 \big(\underbrace{-9\cos^2 t - 9\sin^2 t}_{-9}\big)\, ds\,dt$$

$$= 2\pi\cdot 4\cdot -9 = -72\pi.$$

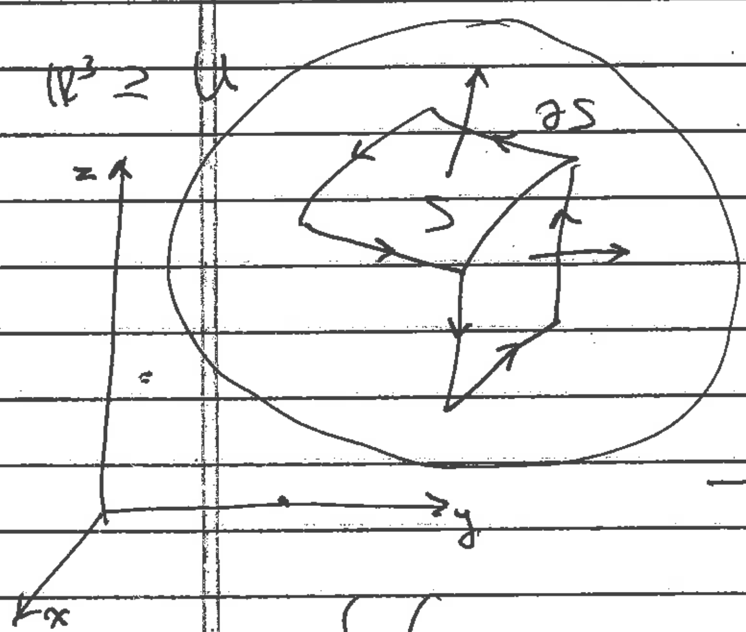
$$\iint_S \vec{F}\cdot d\vec{S} = \left(+\iint_X - \iint_Y - \iint_Z\right)\vec{F}\cdot d\vec{S}$$

$$= +0 - 0 - (-72\pi) = \boxed{72\pi.}$$

7.3

STOKES' THM

Let S be a bounded piecewise smooth oriented 2-surface in $\mathbb{R}^3$. Suppose the boundary ∂S of S consists of finitely many piecewise diff'ble (continuously) curves, oriented consistently with S.



Let $F: U^{open} \subseteq \mathbb{R}^3 \to \mathbb{R}^3$ be C^1 vector field, $S \subseteq U \subseteq \mathbb{R}^3$.

THEN:

$$\iint_S \underbrace{(\nabla \times \vec{F})}_{\text{curl } F} \cdot d\vec{S} = \oint_{\partial S} \vec{F} \cdot d\vec{s}$$

7.3 Exc #4 p 505.

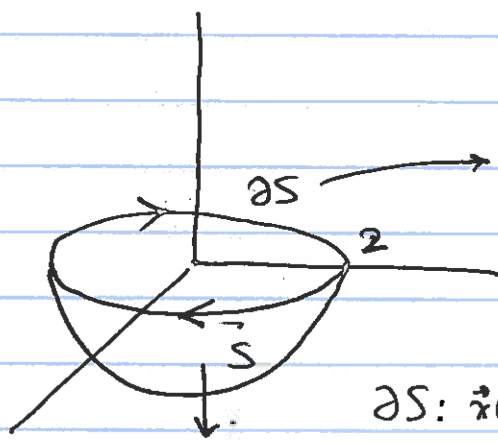
Verify Stokes' Thm $\rightarrow \oint_{\partial S} F \cdot ds$, $\rightarrow \iint \text{curl}\vec{F} \cdot \vec{dS}$ } check $=$ or not.

$$F = (2y - z,\ x + y^2 - z,\ 4y - 3x)$$

S $\quad x^2 + y^2 + z^2 = 4$

$z \le 0$

Downward normal.



$\partial S \rightarrow$ circle, on xy plane radius 2

$$\partial S: \ \vec{x}(t) = (\overset{x}{\overbrace{2\sin t}},\ \overset{y}{\overbrace{2\cos t}},\ \overset{z}{\overbrace{0}})$$

$$0 \le t \le 2\pi.$$

$$\vec{x}'(t) = (+2\cos t, -2\sin t, 0)$$

$$\oint_{C = \partial S} \vec{F} \cdot \vec{ds} = \int_0^{2\pi} \vec{F}(x(t)) \cdot \vec{x}'(t)\, dt$$

$$= \int_0^{2\pi} (4\cos t - 0,\ 2\sin t + 4\cos^2 t - 0,\ 8\cos t - 6\sin t) \cdot (2\cos t, -2\sin t, 0)\, dt$$

$$= \int_0^{2\pi} (8\cos^2 t - 4\sin^2 t - 8\cos^2 t \sin t)\, dt$$

$$\oint_{\partial S} \vec{F}\cdot d\vec{s} = \int_0^{2\pi} \left(8\cdot \frac{1+\cos 2t}{2} - 4\cdot \frac{1-\cos 2t}{2} + 8\cos^2 t \sin t \right) dt$$

$$\oint_{\partial S} \vec{F}\cdot d\vec{s} = (4-2)\cdot 2\pi = 2\pi\cdot 2 = 4\pi, \quad \text{since}$$

$$\begin{cases} \displaystyle\int_0^{2\pi} \cos^2 t \sin t \, dt = -\frac{\cos^3 t}{3}\Big|_0^{2\pi} = 0. \\ \qquad u = \cos t \\ \qquad du = -\sin t \, dt \\ \displaystyle\int_0^{2\pi} \cos 2t \, dt = 0. \end{cases}$$

<u>Next page</u>: $\iint \text{curl } F \cdot dS$

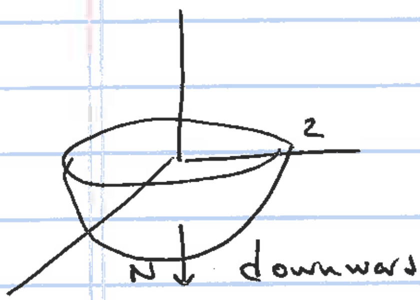
First Find Curl F

$$\nabla \times F = \text{curl } F = \begin{vmatrix} i & j & k \\ \partial_x & \partial_y & \partial_z \\ 2y-z & x+y^2-z & 4y-3x \end{vmatrix}$$

$$= (4-(-1), -1-(-3), 1-2)$$

$$= (5, 2, -1)$$

$$\iint_S \underbrace{\overrightarrow{\text{curl } F}}_{\nabla \times F} \cdot \overrightarrow{dS} = \iint_S (5, 2, -1) \cdot \overrightarrow{dS}$$



$$\underline{X}(s,t) = \left(s, t, -\sqrt{4-s^2-t^2}\right)$$

$$\underline{X}_s = \left(1, 0, -\frac{-2s}{2\sqrt{4-s^2-t^2}}\right)$$

$$\underline{X}_t = \left(0, 1, -\frac{-2t}{2\sqrt{4-s^2-t^2}}\right)$$

$$N = \underline{X}_s \times \underline{X}_t = \left(\frac{-s}{\sqrt{4-s^2-t^2}}, \frac{-t}{\sqrt{4-s^2-t^2}}, 1\right)$$

upward

opposite normals

$$\iint_S \overrightarrow{\nabla \times F} \cdot \overrightarrow{dS} = -\iint (5, 2, -1) \cdot \left(\frac{-s}{\sqrt{}}, \frac{-t}{\sqrt{}}, 1\right) ds\,dt$$

$$D = \{(s,t) \mid s^2+t^2 \leq 4\}$$

$$= -\iint_{s^2+t^2 \leq 4} \left(\frac{-5s-2t}{\sqrt{4-s^2-t^2}} - 1\right) ds\,dt$$

$$= \underbrace{\iint_{s^2+t^2\leq 4} \frac{5s+2t}{\sqrt{4-s^2-t^2}}}_{\text{PTO} \quad 0} + \underbrace{\iint_{s^2+t^2 \leq 4} 1 \, ds\,dt}_{4\pi} = 4\pi$$

$$\oint_{\partial S} \overrightarrow{F} \cdot \overrightarrow{ds} = 4\pi = \iint_S \overrightarrow{\text{curl } F} \cdot \overrightarrow{dS} \checkmark$$

Use polar coordinates

$$\iint_{s^2+t^2\le 4} \frac{s}{\sqrt{4-s^2-t^2}}\,ds\,dt = \int_0^{2\pi}\int_0^2 \frac{r\cos\theta}{\sqrt{4-r^2}}\,r\,dr\,d\theta$$

$$= \underbrace{\left(\int_0^2 \frac{r^2\,dr}{\sqrt{4-r^2}}\right)}_{\text{improper but converges}} \underbrace{\left(\int_0^{2\pi} \cos\theta\,d\theta\right)}_{0} = 0.$$