May 1, 2017

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Final Exam covers:
Chap 5-6-7.

5.1-4	1
5.5	2
6.1	1
6.2	1
6.3	1
7.1	1
7.2	1
7.3	2

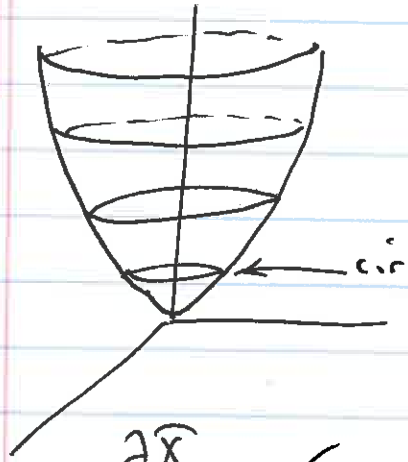
Review Session Tentatively Tuesday May 9 Evening

⑦.① p 468 #7

$$\mathbf{X}(s,t) = (s\cos t,\ s\sin t,\ s^2)$$

$0 \leq t \leq 2\pi$

$0 \leq s$



circle of radius s at height s^2.

i j k

$$\frac{\partial \mathbf{X}}{\partial s} = (\cos t,\ \sin t,\ 2s)$$

$$\frac{\partial \mathbf{X}}{\partial t} = (-s\sin t,\ s\cos t,\ 0)$$

$$N = (-2s^2\cos t,\ -2s^2\sin t,\ s)$$

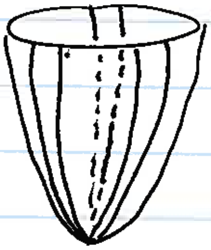
$$= s(-2s\cos t,\ -2s\sin t,\ 1) \neq 0 \text{ we want}$$

S is smooth if $s > 0$.

Coordinate curves

$s = s_0$

$t = t_0$



7.1 p.468 #17 Continue

Eqn of the tangent plane to $\underline{X}(s,t)$ at the point $(1,\sqrt{3},4)$

$\underline{X}(s,t) = (s\cos t, s\sin t, s^2)$ for which (s_0,t_0) Do I get $(1,\sqrt{3},4)$

$N(s,t) = s(-2s\cos t, -2s\sin t, 1)$

Tangent plane $N(s_0,t_0)\cdot\left[(x,y,z) - \underline{X}(s_0,t_0)\right] = 0$

$$\underline{X}\Big(\overset{\overset{s_0}{\shortparallel}}{2}, \overset{\overset{t_0}{\shortparallel}}{\tfrac{\pi}{3}}\Big) = (1,\sqrt{3},4)$$

$$N\left(2,\frac{\pi}{3}\right) = 2\left(-4\cdot\frac{1}{2}, -4\cdot\frac{\sqrt{3}}{2}, 1\right)$$

$$= (-4, -4\sqrt{3}, 2)$$

$$(-4,-4\sqrt{3},2)\cdot\left[(x,y,z) - (1,\sqrt{3},4)\right] = 0.$$

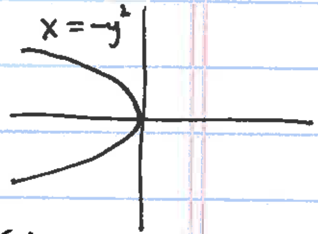
$$-4x - 4\sqrt{3}y + 2z = (-4,-4\sqrt{3},2)\cdot(1,\sqrt{3},4)$$

$$= -4 - 12 + 8 = -8$$

$$-4x - 4\sqrt{3}y + 2z = -8$$

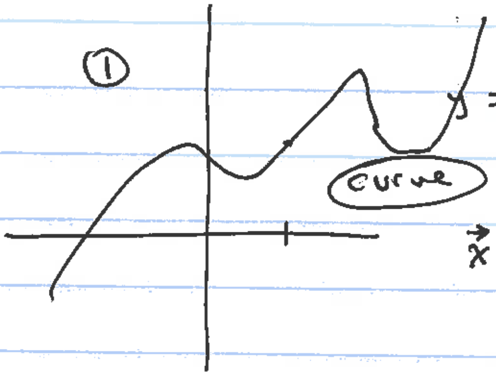
How to parametrize graphs of functions

② $x = -y^2$



$\vec{x}(t) = (-t^2, t)$

(6.1) ①



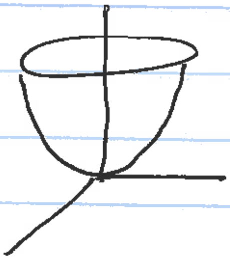
$y = f(x) = x^5 + x^4 - x^3 + x^2 + 1$

curve

$\vec{x}(t) = (\underset{x}{\underset{\parallel}{t}}, \underset{y}{\underset{\parallel}{f(t)}})$

(7.1) ① $z = x^2 + y^2$

surface



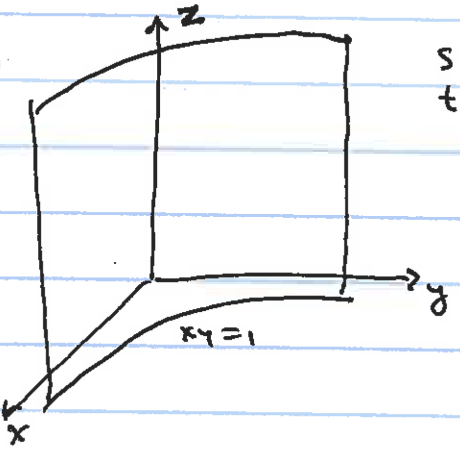
$$\vec{X}(s,t) = (\overset{x}{\overset{\parallel}{s}}, \overset{y}{\overset{\parallel}{t}}, \overbrace{s^2+t^2}^{f(x,y)=f(s,t)})$$

$$\frac{\partial \vec{X}}{\partial s} = (1, 0, 2s)$$

$$\frac{\partial \vec{X}}{\partial t} = (0, 1, 2t)$$

$$N = \frac{\partial \vec{X}}{\partial s} \times \frac{\partial \vec{X}}{\partial t} = (-2s, -2t, 1) \neq 0 \quad \forall s,t.$$

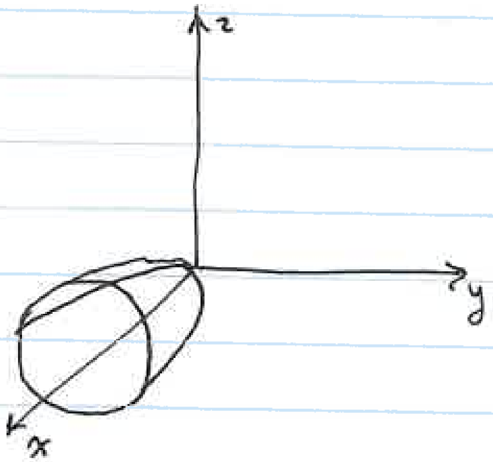
② $\vec{X}(s,t) = (\underset{x}{\underset{\parallel}{s}}, \underset{y}{\underset{\parallel}{\frac{1}{s}}}, t)$



$s > 0$
$t \geq 0$

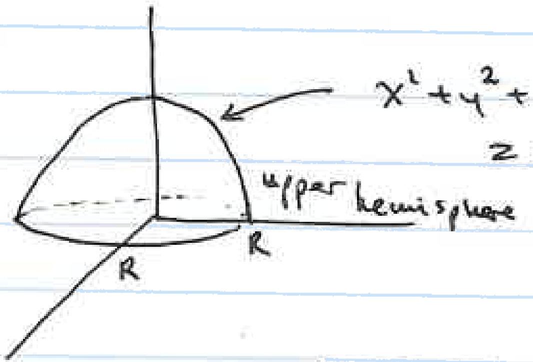
③ $\big(\underbrace{s^2+t^2}_{x},\ \underset{y}{\underset{\shortparallel}{s}},\ \underset{z}{\underset{\shortparallel}{t}}\big)$

parametrizes graph of $x = y^2+z^2$



Parametrizing Spheres:

①



$x^2+y^2+z^2=R^2$

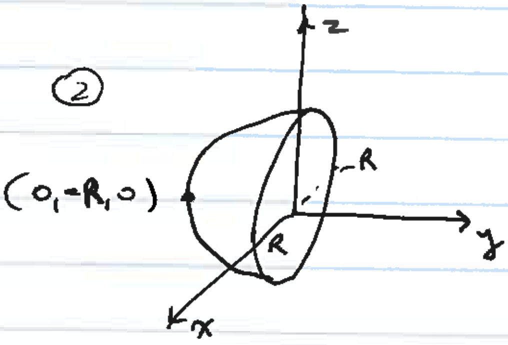
$z = \pm\sqrt{R^2-x^2-y^2}$

$$\Phi(s,t) = \big(\underset{x}{\underset{\shortparallel}{s}},\ \underset{y}{\underset{\shortparallel}{t}},\ \sqrt{R^2-s^2-t^2}\big)$$

<u>need:</u> $s^2+t^2 < R^2$

$$\Psi(s,t) = (s,\ -\sqrt{R^2-s^2-t^2},\ t)$$

②

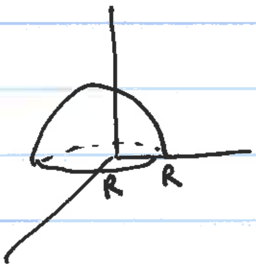


③ $Z(r,\theta) = \left(r\cos\theta,\ r\sin\theta,\ \sqrt{R^2 - r^2}\right)$

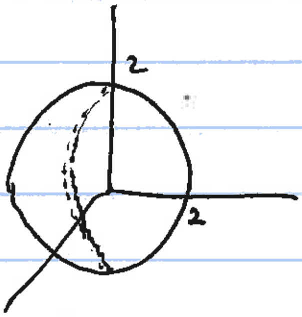
$0 \le \theta \le 2\pi$

$0 < r$

sphere by using cylindrical coordinates



④ sphere by using spherical coordinates



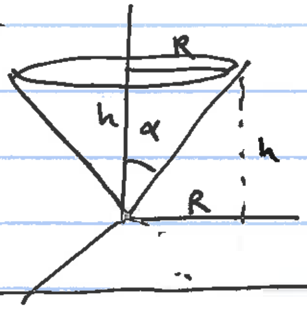
radius

$W(\theta,\varphi) = \left(2\sin\varphi\cos\theta,\ 2\sin\varphi\sin\theta,\ 2\cos\varphi\right)$

$0 \le \theta \le 2\pi$

$0 \le \varphi \le \pi$

Ex Cone



$Y(s,t) = \left(\overset{x}{\overset{\|}{s}},\ \overset{y}{\overset{\|}{t}},\ \frac{h}{R}\sqrt{s^2+t^2}\right)$

OR

$X(s,t) = \left(\underbrace{s\cos t}_{x},\ \underbrace{s\sin t}_{y},\ \frac{h}{R}s\right)$

$= \cot\alpha = h/R$

Remark: $(s\cos t,\ s\sin t,\ C)$ parametrizes the plane $z = C$.

Must know how to parametrize

- Planes
- Graphs, $z = f(x,y)$; $x = g(y,z)$; $y = h(x,z)$
- Cylinders
- Spheres
- Cones

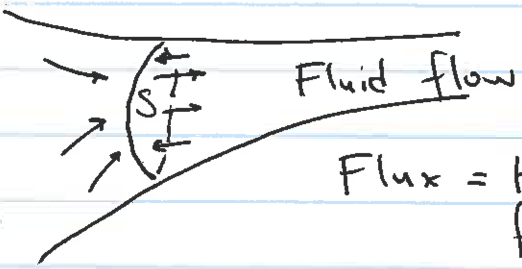
(7.2) Surface Integrals

Will Do ① Area of a surface

(Skip) not needed in 7.3 ② $\iint f \cdot dS$ f real valued (mass of a metallic bowl with variable density f)

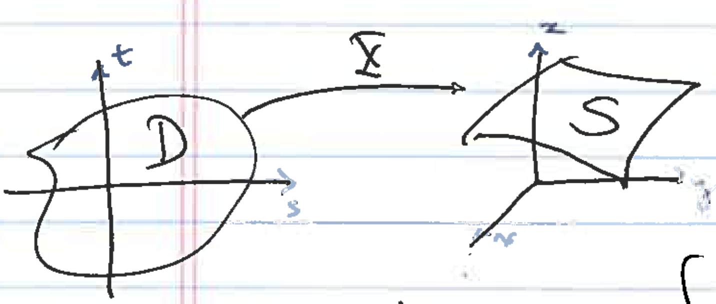
Will Do ③ $\iint_S \vec{F} \cdot d\vec{S}$ Flux integrals

Need it in 7.3



Flux = How much total fluid passes from one side to the other thru S

Defn Let $\underline{X}(s,t) : D \longrightarrow \mathbb{R}^3$ be a parametrized smooth surface, Let $\underline{X}$ be 1-1, onto S



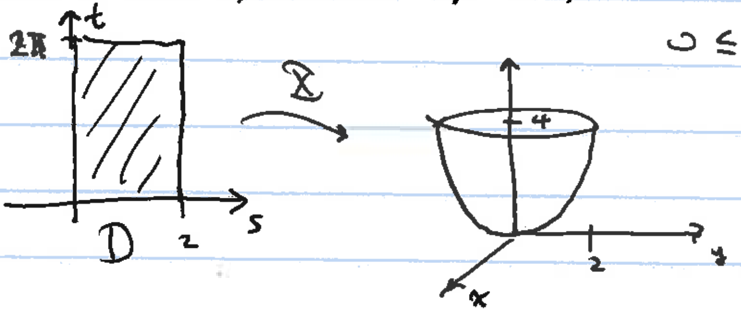
① Area S = $\iint_D \|N\| \, ds \, dt$

② $\iint_S f \cdot dS = \iint_D f(\underline{X}(s,t)) \, \|N\| \, ds \, dt$

where $N = \frac{\partial \underline{X}}{\partial s} \times \frac{\partial \underline{X}}{\partial t}$

Recall
$$\begin{cases} \dfrac{\partial \mathbb{X}}{\partial s}, \dfrac{\partial \mathbb{X}}{\partial t} \\ N = \dfrac{\partial \mathbb{X}}{\partial s} \times \dfrac{\partial \mathbb{X}}{\partial t} \end{cases}$$
$$dS = \|N\|\, ds\, dt$$

Ex $\mathbb{X}(s,t) = (s\cos t,\ s\sin t,\ s^2)$ $0 \le s \le 2$, $0 \le t \le 2\pi$



- Area S
- $\iint_S x^2 dS$ (if time)

$$\begin{array}{cccc} & i & j & k \\ \dfrac{\partial \mathbb{X}}{\partial s} = & (\cos t, & \sin t, & 2s) \end{array}$$

$$\frac{\partial \mathbb{X}}{\partial t} = (-s\sin t,\ s\cos t,\ 0)$$

$$\frac{\partial \mathbb{X}}{\partial s} \times \frac{\partial \mathbb{X}}{\partial t} = N = (-2s^2\cos t,\ -2s^2 \sin t,\ s)$$

$$\|N\| = \sqrt{4s^4\cos^2 t + 4s^4\sin^2 t + s^2}$$

$$= \sqrt{4s^4 + s^2}$$

$$\text{Area} = \iint_D \|N\|\, ds\, dt = \int_0^2 \int_0^{2\pi} \sqrt{4s^4 + s^2}\ dt\, ds$$

$s > 0$

$$= \int_0^2 \int_0^{2\pi} \sqrt{s^2 (4s^2+1)} \, dt \, ds$$

$$= \int_0^2 \int_0^{2\pi} \sqrt{4s^2+1} \, dt \; s\,ds .$$

$u = 4s^2+1$

$du = 8s\,ds$

$$= \left(\int_0^{2\pi} dt \right) \left(\int_1^{17} \underbrace{\sqrt{u}}_{\int u^{1/2}} \cdot \frac{1}{8} \, du \right)$$

$$= 2\pi \cdot \frac{1}{8} \; u^{3/2} \cdot \frac{2}{3} \Big|_1^{17}$$

$$= \frac{\pi}{6} \left(17\sqrt{17} - 1 \right)$$

If you are curious about it: (NOT IN the final)

$$\iint_S x^2 \, dS = \int_0^2 \int_0^{2\pi} \underbrace{(s \cdot \cos t)^2}_{x^2} \cdot \underbrace{\sqrt{4s^4 + s^2}}_{|N|} \, ds \, dt$$

$$\mathbf{X}(s,t) = (\underbrace{s\cos t}_{x}, s\sin t, s^2)$$

$$= \int_0^2 s^2 \cdot s \cdot \sqrt{4s^2+1} \, ds \cdot \int_0^{2\pi} \cos^2 t \, dt$$

$u = 4s^2+1$

$du = 8s\,ds$

$s^2 = \frac{u-1}{4}$

$$= \int_1^{17} \frac{u-1}{4} \cdot \sqrt{u} \cdot \frac{1}{8} \, du \int_0^{2\pi} \frac{1+\cos 2t}{2} \, dt = \frac{\pi}{32} \left(\frac{2}{5} 17^2 \sqrt{17} - \frac{2}{5} - \frac{2}{3} 17\sqrt{17} + \frac{2}{3} \right).$$