

Some HW problems

April 13

(5.4) Exc #15 p 348

(1)

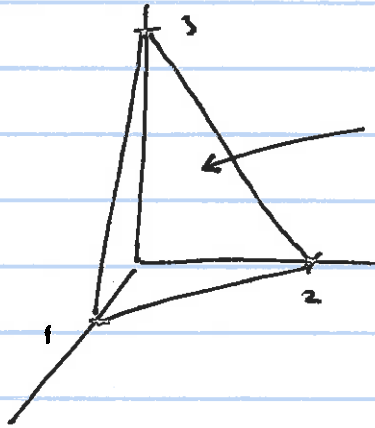
$$f = 1 - z^2$$

$$(0, 0, 0)$$

$$(1, 0, 0)$$

$$(0, 2, 0)$$

$$(0, 0, 3)$$



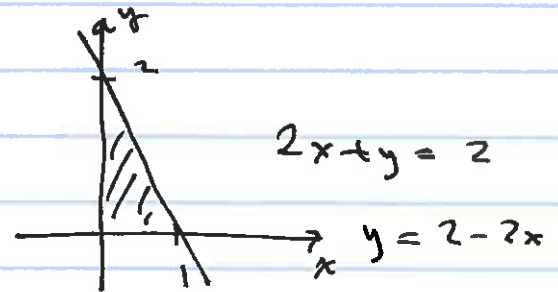
$$6x + 3y + 2z = 6$$

$$z = \frac{6 - 6x - 3y}{2}$$

$$x = \frac{6 - 3y - 2z}{6}$$

$$\int_0^1 \int_0^{2-2x} \int_0^{3-3x-\frac{3}{2}y} (1 - z^2) dz dy dx$$

$$\left. \begin{aligned} 0 \leq x \leq 1 \\ 0 \leq y \leq 2 - 2x \\ 0 \leq z \leq \frac{6 - 6x - 3y}{2} \end{aligned} \right\}$$

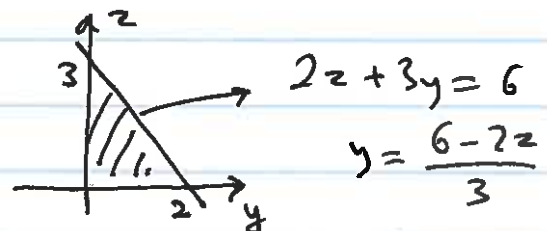


$$I = \int_0^1 \int_0^{2-2x} \int_0^{3-3x-\frac{3}{2}y} (1 - z^2) dz dy dx \quad / \text{too long}$$

Use another order

$$\int_0^3 \int_0^{\frac{6-2z}{3}} \int_0^{1-\frac{1}{2}y-\frac{1}{3}z} (1 - z^2) dx dy dz$$

$$\left. \begin{aligned} 0 \leq z \leq 3 \\ 0 \leq y \leq \frac{6 - 2z}{3} \end{aligned} \right\}$$



(2)

$$= \int_0^3 \int_0^{2-\frac{2}{3}z} (1-z^2) x \Big|_{x=0}^{x=1-\frac{1}{2}y-\frac{1}{3}z} dy dz$$

$$= \int_0^3 \int_0^{2-\frac{2}{3}z} (1-z^2) (1-\frac{1}{2}y-\frac{1}{3}z) dy dz.$$

$$= \int_0^3 (1-z^2) \left(y - \frac{y^2}{4} - \frac{1}{3}yz \right) \Big|_{y=0}^{y=2-\frac{2}{3}z} dz.$$

$$= \int_0^3 (1-z^2) \left(\underbrace{2-\frac{2}{3}z}_y \right) \left(1 - \frac{2-\frac{2}{3}z}{4} - \frac{1}{3}z \right) dz.$$

$$\frac{1}{3}(6-2z) \quad \underbrace{1 - \frac{1}{2} + \frac{1}{6}z - \frac{1}{3}z}_{\frac{1}{2} - \frac{1}{6}z}.$$

$$\frac{1}{6}(3-z)$$

$$= \int_0^3 \frac{1}{18} (1-z^2) \underbrace{(6-2z)}_{2(3-z)} (3-z) dz$$

$$= \int_0^3 \frac{1}{9} (1-z^2) (3-z)^2 dz.$$

$$\frac{1}{9} (1-z^2) (9-6z+z^2)$$

$$\frac{1}{9} (9 + 6z + z^2 - 9z^2 - 6z^3 - z^4)$$

$$I = \int_0^3 \frac{1}{9} (9 - 6z - 8z^2 + 6z^3 - z^4) dz \quad (3)$$

$$= \frac{1}{9} \left(9z - 3z^2 - \frac{8}{3}z^3 + \frac{3}{2}z^4 - \frac{z^5}{5} \right) \Big|_0^3$$

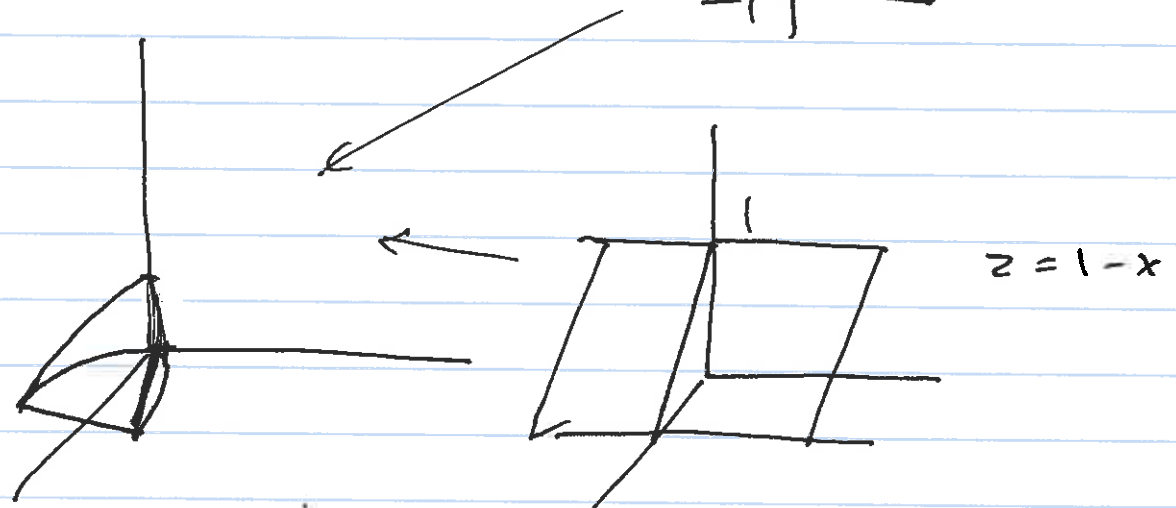
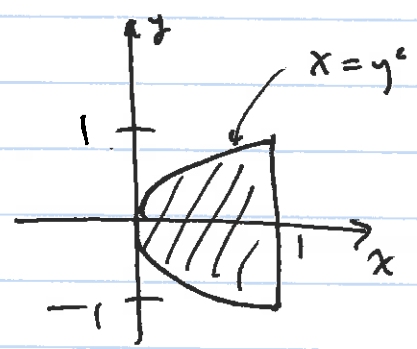
$$= \frac{1}{9} \left(\cancel{27} - \cancel{27} - 72 + \frac{243}{2} - \frac{243}{5} \right)$$

$$= \frac{1}{9} \left(\frac{-720 + 1215 - 486}{10} \right) = \frac{1}{9} \cdot \frac{9}{10} = \frac{1}{10}$$

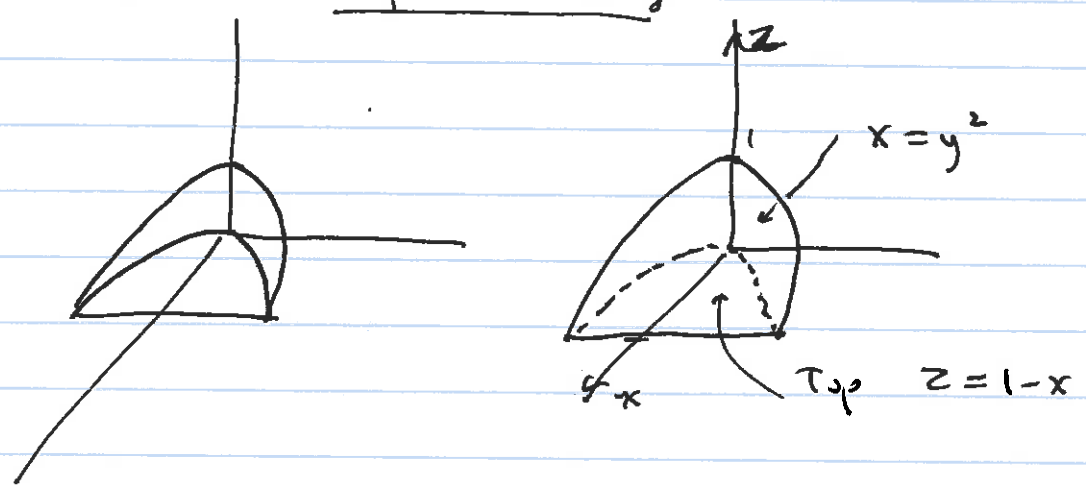
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$$\int_{-1}^1 \int_{y^2}^1 \int_0^{1-x} f \, dz \, dx \, dy$$

$$\begin{cases} -1 \leq y \leq 1 \\ 1 \leq x \leq y^2 \\ 0 \leq z \leq 1-x \end{cases}$$

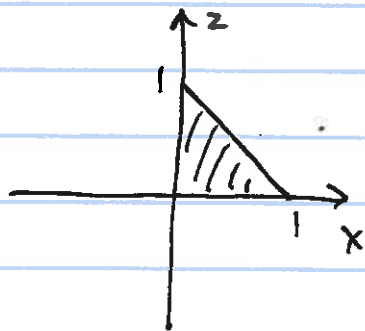


improve drawing:



$$\int_0^1 \int_{-\sqrt{x}}^{\sqrt{x}} \int_0^{1-x} f \, dz \, dy \, dx$$

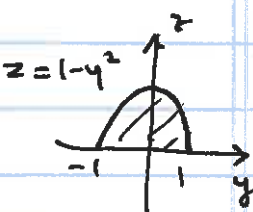
view from y-axis direction



$$\int_0^1 \int_0^{1-z} \int_{-\sqrt{x}}^{\sqrt{x}} f \, dy \, dx \, dz$$

$$\int_0^1 \int_0^{1-x} \int_{-\sqrt{x}}^{\sqrt{x}} f \, dy \, dz \, dx$$

$$y = \pm\sqrt{1-z}$$



x-axis view

$$\int_0^1 \int_{-\sqrt{1-z}}^{\sqrt{1-z}} \int_{y^2}^{1-z} f \, dx \, dy \, dz$$

$$\int_{-1}^1 \int_0^{1-y^2} \int_{y^2}^{1-z} f \, dx \, dz \, dy$$