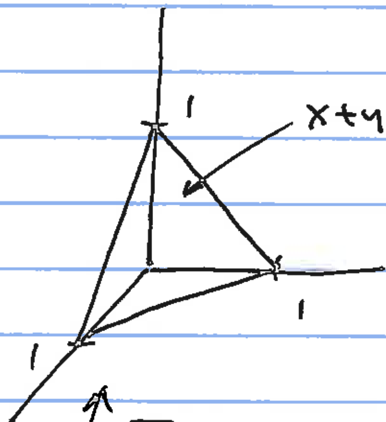
April 6

(1)

5.4 Examples

Recall $\displaystyle\iiint_D 1\, dV = \text{volume}(D)$ $\quad D \subseteq \mathbb{R}^3$

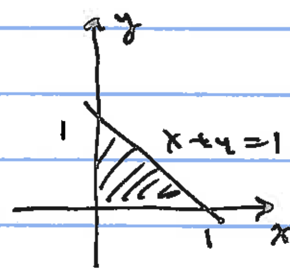


Find Volume.

Tetrahedron in 1^{st} Octant

$x, y, z \geq 0$

Top view



$$\begin{cases} 0 \le x \le 1 \\ 0 \le y \le 1-x \end{cases}$$

$0 \le x \le 1$

$0 \le y \le 1-x$

$0 \le z \le 1-x-y$

$$V = \int_0^1 \int_0^{1-x} \int_0^{1-x-y} 1 \; dz\, dy\, dx$$

$$= \int_0^1 \int_0^{1-x} z \Big|_{z=0}^{z=1-x-y} dy\, dx$$

$$= \int_0^1 \int_0^{1-x} (1-x-y) \; dy\, dx$$

$$= \int_0^1 (1-x)y - \frac{y^2}{2} \Big|_0^{1-x} dx$$

$$= \int_0^1 (1-x)(1-x) - \frac{(1-x)^2}{2} dx$$

$$= \int_0^1 \frac{(1-x)^2}{2} dx = \int_1^0 (-du) \frac{u^2}{2}$$

$u = 1-x$

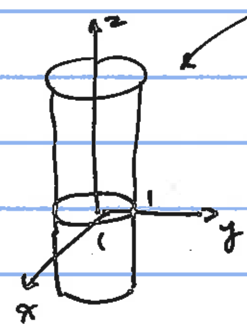
$du = -dx$

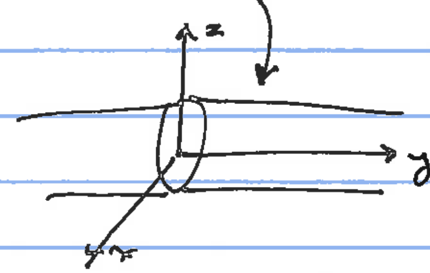
$$= \frac{u^3}{6} \Big|_0^1 = \frac{1}{6} - 0 = \frac{1}{6}.$$

Exc #24 Volume of the region inside two cylinders

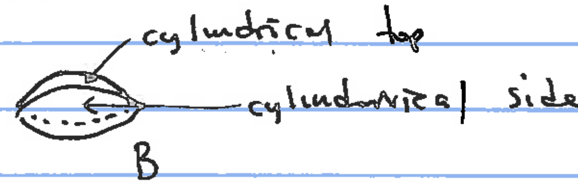
$x^2+y^2=1$

$x^2+z^2=1$

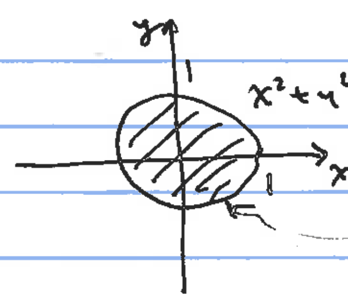




Round ball $\neq$



Top view



$x^2+y^2=1.$

$-1 \le x \le 1$

$-\sqrt{1-x^2} \le y \le \sqrt{1-x^2}$

B: $-1 \le x \le 1$

$$-\sqrt{1-x^2} \le y \le \sqrt{1-x^2}$$

$$-\sqrt{1-x^2} \le z \le \sqrt{1-x^2}$$

$$\int_{-1}^{1} \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} 1 \, dz \, dy \, dx.$$

OR

$$-1 \le y \le 1$$

$$-\sqrt{1-y^2} \le x \le \sqrt{1-y^2}$$

$$-\sqrt{1-x^2} \le z \le \sqrt{1-x^2}$$

$$\int_{-1}^{1} \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} 1 \, dz \, dx \, dy$$

which one is easier

D. $\displaystyle\int_{-1}^{1} \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} z \Big|_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} dy \, dx$

$$= \int_{-1}^{1} \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} 2\sqrt{1-x^2} \, dy \, dx$$

$$= \int_{-1}^{1} 2y\sqrt{1-x^2} \Big|_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} dx$$

$$= \int_{-1}^{1} 4(1-x^2) \, dx = 4\left(x - \frac{x^3}{3} \Big|_{-1}^{1}\right) = \frac{16}{3}.$$

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§ 6.4 #26

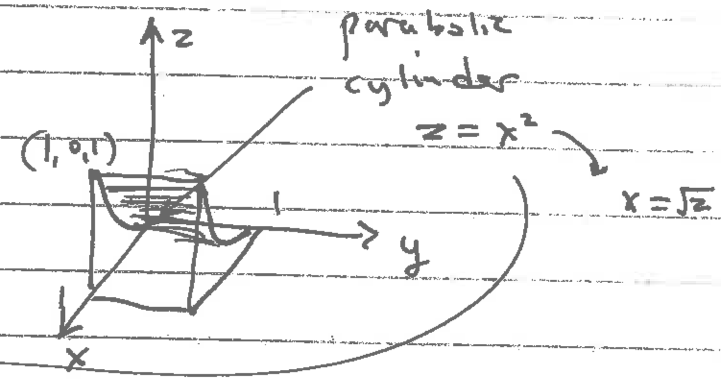
$$\int_0^1 \int_0^1 \int_0^{x^2} f \, dz \, dy \, dx.$$

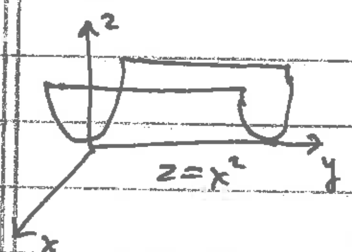
Change the order of integration

$0 \le x \le 1$

$0 \le y \le 1$

$0 \le z \le x^2$

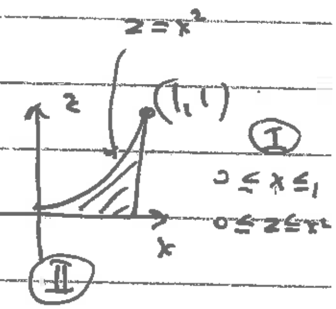




$$\int_0^1 \int_0^1 \int_0^{x^2} f \, dz \, dx \, dy$$

$$\int_0^1 \int_0^{x^2} \int_0^1 f \, dy \, dz \, dx$$

$$\int_0^1 \int_{\sqrt{z}}^1 \int_0^1 f \, dy \, dx \, dz$$

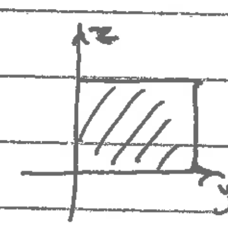


I: $0 \le x \le 1$, $0 \le z \le x^2$

II: $0 \le z \le 1$, $\sqrt{z} \le x \le 1$

$$\int_0^1 \int_0^1 \int_{\sqrt{z}}^1 f \, dx \, dy \, dz$$

$$\int_0^1 \int_0^1 \int_{\sqrt{z}}^1 f \, dx \, dz \, dy$$



$0 \le y \le 1$

$0 \le z \le 1$