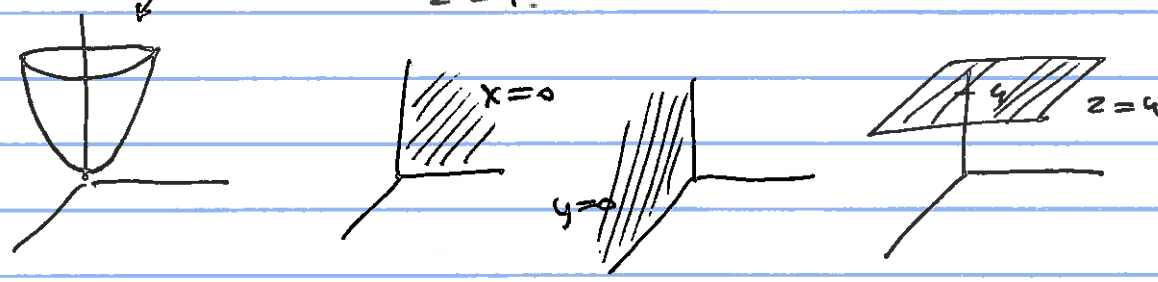
#16 p348 First octant $x, y, z \geq 0$

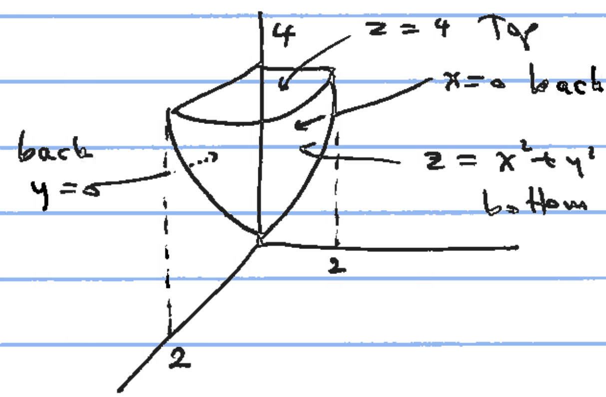
$z = x^2 + y^2$

$x = 0$

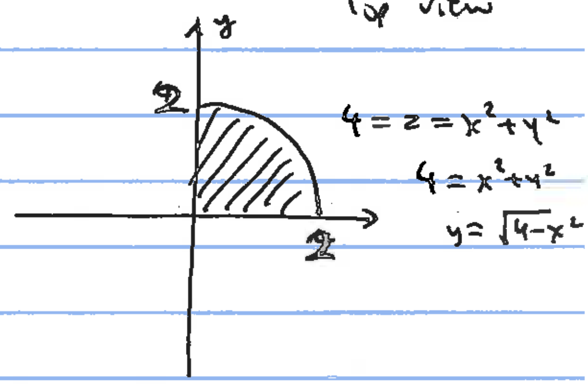
$y = 0$

$z = 4$.





Top view



$0 \leq x \leq 2$

$0 \leq y \leq \sqrt{4-x^2}$

$$\int_0^2 \int_0^{\sqrt{4-x^2}} \int_{x^2+y^2}^{4} 3x \quad dz \; dy \; dx$$

To be continued.

$$= \int_0^2 \int_0^{\sqrt{4-x^2}} \int_{x^2+y^2}^4 3x \quad dz\, dy\, dx$$

$$= \int_0^2 \int_0^{\sqrt{4-x^2}} 3xz \Big]_{z=x^2+y^2}^{z=4} \quad dy\, dx$$

$$= \int_0^2 \int_0^{\sqrt{4-x^2}} 12x - 3x\,(x^2+y^2)\; dy\; dx$$

$$= \int_0^2 \int_0^{\sqrt{4-x^2}} 12x - 3x^3 - 3xy^2 \;\; dy\; dx$$

$$= \int_0^2 12xy - 3yx^3 - xy^3 \Big|_{y=0}^{y=\sqrt{4-x^2}} \quad dx$$

$$= \int_0^2 12x\sqrt{4-x^2} - 3\sqrt{4-x^2}\cdot x^3 - x\left(\sqrt{4-x^2}\right)^3 \; dx$$

$$= \int_0^2 \sqrt{4-x^2}\,\left[\,12x - 3x^3 - 4x + x^3\,\right] dx$$

$$= \int_0^2 \sqrt{4-x^2}\,\underbrace{\left[\,8x - 2x^3\right]}_{2x(4-x^2)}\; dx$$

$u = 4 - x^2$

$du = -2x\,dx.$

$$= \int_4^0 (-du)\ u^{\frac{1}{2}} \cdot u$$

$$= \int_0^4 u^{3/2}\, du. \quad = \quad \frac{2}{5} u^{5/2} \Big|_0^4$$

$$= \frac{2}{5}\left(4^{\frac{5}{2}} - 0^{\frac{5}{2}} \right) = \frac{2}{5} \cdot (32 - 0) = \frac{64}{5}.$$

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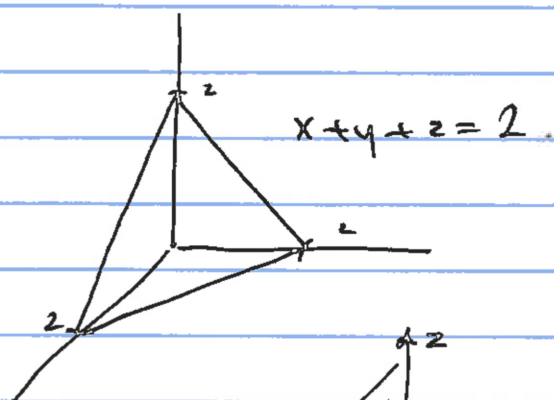
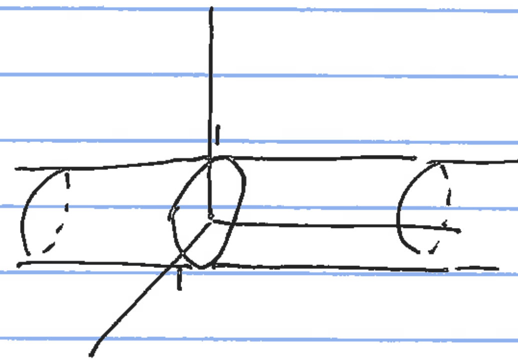
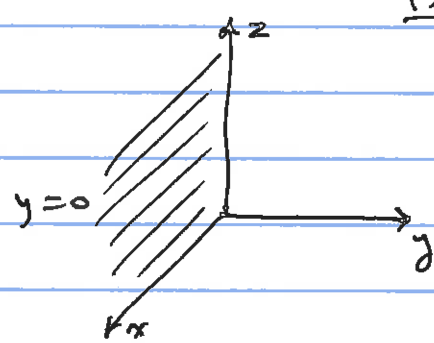
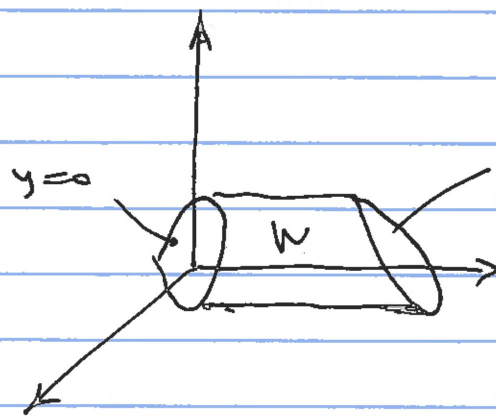
$f = xy$

W bdd by $\begin{cases} x+y+z=2 \\ x^2+z^2=1 \\ y=0. \end{cases}$







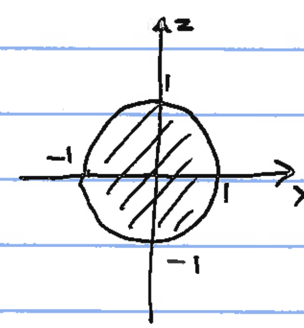


$x+y+z=2$

$y = 2-z-x.$

view from y-axis direction

$$\left.\begin{array}{c} -1 \leq x \leq 1 \\ -\sqrt{1-x^2} \leq z \leq \sqrt{1-x^2} \\ 0 \leq y \leq 2-z-x. \end{array}\right\} \longrightarrow$$



$$I = \int_{-1}^{1} \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \int_{0}^{2-z-x} y \quad dy\, dz\, dx$$

$$= \int_{-1}^{1} \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \left(\frac{y^2}{2} \Big|_{y=0}^{y=2-z-x} \right) dz\, dx$$

$$= \int_{-1}^{1} \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \frac{1}{2} \left((2-z-x)^2 - 0 \right) dz\, dx$$

$$= \int_{-1}^{1} \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \frac{1}{2} \left(4 + z^2 + x^2 - 4z - 4x + 2xz \right) dz\, dx$$

$$= \int_{-1}^{1} \frac{1}{2} \left(4z + \frac{z^3}{3} + x^2 z - 2z^2 - 4xz + xz^2 \Big|_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \right. dx$$

$$= \int_{-1}^{1} \frac{1}{2} \left(8\sqrt{1-x^2} + \frac{2}{3}\left(\sqrt{1-x^2}\right)^3 + 2x^2\sqrt{1-x^2} - 0 - 8x\sqrt{1-x^2} + 0 \right) dx$$

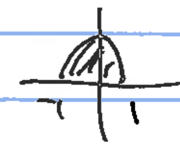
$$= \int_{-1}^{1} \frac{1}{2} \sqrt{1-x^2} \left(8 + \frac{2}{3}(1-x^2) + 2x^2 - 8x \right) dx$$

$$= \int_{-1}^{1} \frac{1}{2} \sqrt{1-x^2} \left(\frac{26}{3} + \frac{4}{3}x^2 - 8x \right) dx = \int_{-1}^{1} \sqrt{1-x^2} \left(\frac{13}{3} + \frac{2}{3}x^2 - 4x \right) dx$$

$$= \frac{13}{3}\cdot\frac{\pi}{2} + \frac{2}{3}\cdot\frac{\pi}{8} - 4\cdot 0 = \left(\frac{13}{6}+\frac{1}{12}\right)\pi = \frac{9\pi}{4}.$$

Since

$$\int_{-1}^{1}\sqrt{1-x^2}\,dx = \frac{\pi}{2}$$



area of half circle

$$\int_{-1}^{1} x\sqrt{1-x^2}\,dx = 0$$

odd

$$\int_{-1}^{1} x^2\sqrt{1-x^2}\,dx = \int_{-\pi/2}^{\pi/2}(\sin^2 u)\cos u\cdot\cos u\,du$$

$x = \sin u$

$dx = \cos u\,du$

$1-x^2 = 1-\sin^2 u = \cos^2 u$

$$= \int_{-\pi/2}^{\pi/2}\sin^2 u\cos^2 u\,du = \int_{-\pi/2}^{\pi/4}\frac{1}{4}\sin^2 2u\,du$$

$\sin 2u = 2\sin u\cdot\cos u.$

$\sin^2 2u = 4\sin^2 u\cos^2 u$

$$= \int_{-\pi/2}^{\pi/2}\frac{1}{4}\,\frac{1-\cos 4u}{2}\,du = \int_{-\pi/2}^{\pi/2}\left(\frac{1}{8}-\frac{1}{8}\cos 4u\right)du$$

$$= \frac{\pi}{8} + \cancel{\frac{1}{32}\sin 4u}\Big|_{-\pi/2}^{\pi/2} = \frac{\pi}{8}.$$