March 30, 2017

5.2 Exc # 20, p 333.

$$\iint_D 3y \, dA \quad \text{where } D \text{ is bounded by}$$

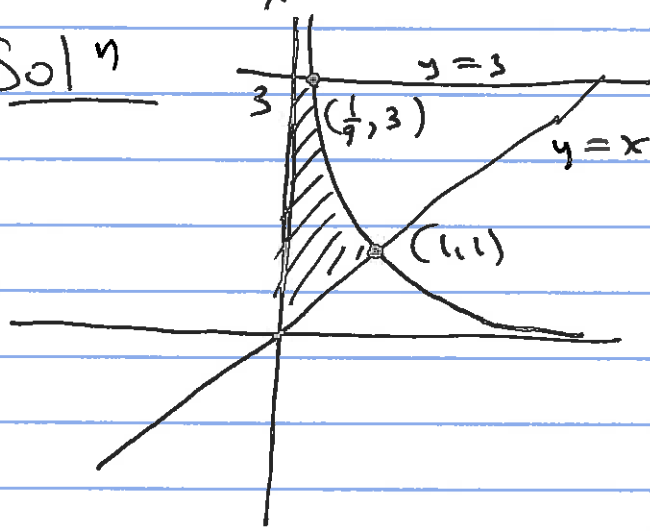
$xy^2 = 1$

$y = x$

$x = 0$

$y = 3.$

Sol^n



Type I

$$\int_0^{1/9} \int_{y=x}^{y=3} 3y \; dy\, dx \;+\; \int_{1/9}^{1} \int_{y=x}^{y=\frac{1}{\sqrt{x}}} 3y \; dy\, dx$$

Type II

$$\left\{ \int_0^1 \int_{x=0}^{x=y} 3y \quad dx\, dy \;+\; \int_1^3 \int_0^{x=\frac{1}{y^2}} 3y \quad dx\, dy \right.$$

We do II

$$\int_0^1 \int_0^y 3y\, dx\, dy + \int_1^3 \int_0^{1/y^2} 3y\, dx\, dy$$

$$= \int_0^1 3yx \Big|_{x=0}^{x=y} dy + \int_1^3 3yx \Big|_{x=0}^{x=1/y^2} dy$$

$$= \int_0^1 (3y^2 - 0)\, dy + \int_1^3 \left(\frac{3}{y} - 0\right) dy$$

$$= y^3 \Big|_{y=0}^{y=1} + 3\ln y \Big|_1^3$$

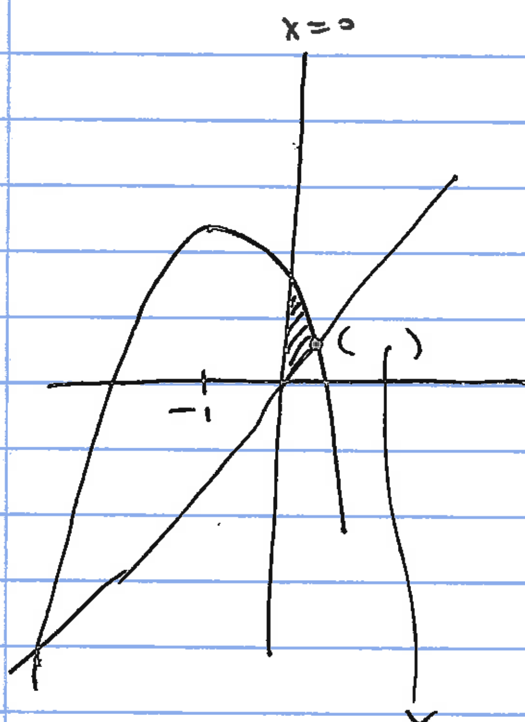
$$0$$

$$= 1 + 3\ln 3 - 3.\cancel{\ln 1}$$

5.2 # 28 Area of D bdd by $\left\{\begin{array}{l} y = 2x \\ x = 0 \\ y = 1 - 2x - x^2 \end{array}\right.$

$+ \underbrace{\text{I}^{st}\text{-quadrant}}$ to make the question well-defined

$= -(x+1)^2 + 2$



$(-2+\sqrt{5}, -4+2\sqrt{5})$

$2x = 1 - 2x - x^2$

$x^2 + 4x - 1 = 0$

$\dfrac{-4 \pm \sqrt{16+4}}{2} = \dfrac{-4 \pm \sqrt{20}}{2}$

$= -2 \pm \sqrt{5}$

$$\text{Area}(D) = \int_0^{\sqrt{5}-2} \int_{2x}^{1-2x-x^2} 1 \, dy \, dx$$

$$= \int_0^{\sqrt{5}-2} y \Big|_{2x}^{1-2x-x^2} dx = \int_0^{\sqrt{5}-2} \left((1-2x-x^2) - 2x \right) dx$$

$$= \int_0^{\sqrt{5}-2} (1 - 4x - x^2) \, dx = x - \frac{4x^2}{2} - \frac{x^3}{3} \Big|_0^{\sqrt{5}-2}$$

$$= (\sqrt{5}-2) - 2(\sqrt{5}-2)^2 - \frac{1}{3}(\sqrt{5}-2)^3 = \cdot$$

$$\boxed{(\sqrt{5}-2)^3 = (\sqrt{5})^3 - 3(\sqrt{5})^2\cdot 2 + 3\sqrt{5}\cdot 2^2 - 2^3}$$

$$I = \sqrt{5} - 2 - 2(5 - 4\sqrt{5} + 4) - \frac{1}{3}(5\sqrt{5} - 30 + 12\sqrt{5} - 8)$$

$$= \sqrt{5} - 2 - 2(9 - 4\sqrt{5}) - \frac{1}{3}(17\sqrt{5} - 38)$$

$$= \sqrt{5} - 2 - 18 + 8\sqrt{5} - \frac{17}{3}\sqrt{5} + \frac{38}{3}$$

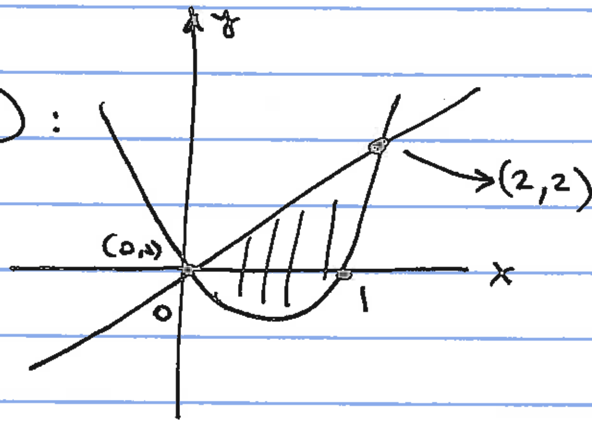
$$= \frac{3\sqrt{5} - 60 + 24\sqrt{5} - 17\sqrt{5} + 38}{3}$$

$$= \frac{-22 + 10\sqrt{5}}{3}$$

Exc #36 5.2

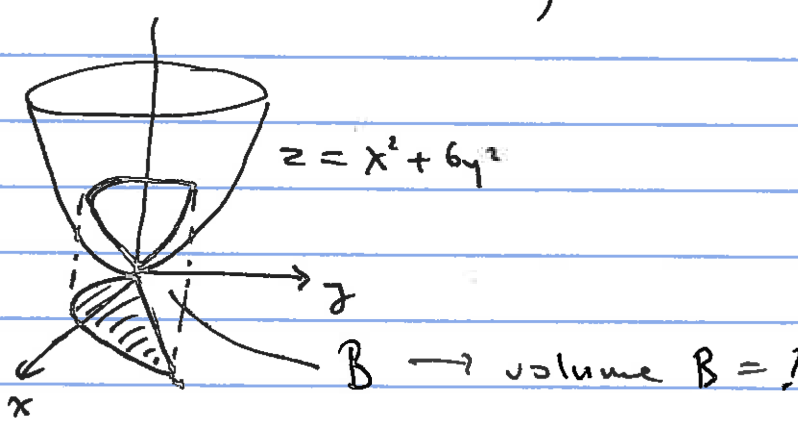
Volume under the graph $z = x^2 + 6y^2$
and above D bdd by $y = x$
$y = x^2 - x$.

D :



$$x = x^2 - x$$
$$0 = x^2 - 2x$$
$$x = 0$$
$$x = 2$$

D: $0 \le x \le 2$
$x^2 - x \le y \le x$.



$$V = \int_0^2 \int_{x^2-x}^{x} (x^2 + 6y^2)\, dy\, dx$$

$$= \int_0^2 x^2 y + 2y^3 \Big|_{x^2-x}^{x} dx$$

6

$$\int_0^2 \left(x^3 + 2x^3\right) - \left(x^2(x^2-x) + 2(x^2-x)^3\right) dx$$

$$\cdots = \int_0^2 \left(-2x^6 + 6x^5 - 7x^4 + 6x^3\right) dx$$

$$= -\frac{2}{7}x^7 + x^6 - \frac{7}{5}x^5 + \frac{6}{4}x^4 \Big|_0^2$$

$$= \cdots = \frac{232}{35}.$$

$$\begin{aligned} & x^3 + 2x^3 - \left(x^4 - x^3 + 2\left((x^2)^3 - 3(x^2)^2 x + 3x^2 \cdot x^2 - x^3\right)\right) \\ &= x^3 + 2x^3 - \left(x^4 - x^3 + 2x^6 - 6x^5 + 6x^4 - 2x^3\right) \\ &= x^3 + 2x^3 - \underline{x^4} + x^3 - \underline{2x^6} + \underline{6x^5} - \underline{6x^4} + 2x^3 \\ &= -2x^6 + 6x^5 - 7x^4 + 6x^3 \end{aligned}$$

$$= -\frac{2}{7}2^7 + 2^6 - \frac{7}{5}2^5 + \frac{3}{2}\cdot 2^4$$

$$= -\frac{256}{7} + 64 - \frac{224}{5} + 24 = 88 - \frac{256 \cdot 5 + 7 \cdot 224}{35}$$

$$= 88 - \frac{1280 + 1568}{35} = \frac{3080 - 1280 - 1568}{35} = \frac{232}{35}$$