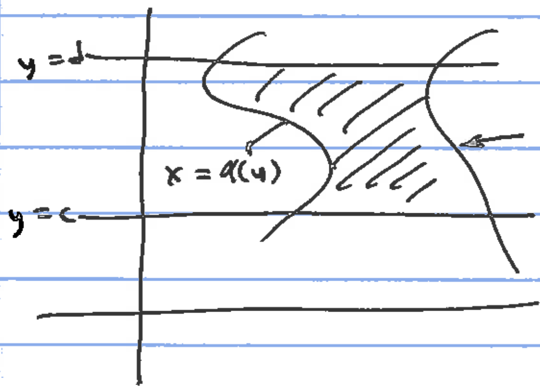
March 29, 2017

5.2

Theorem: Let $f: D \to \mathbb{R}$ be continuous, and D be a type II region



$$D = \{(x,y) \mid c \le y \le d \,;\; \alpha(y) \le x \le \beta(y)\}$$

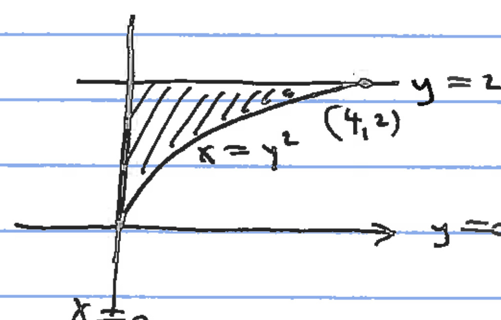
Then

$$\iint_D f\,dA = \int_c^d \int_{\alpha(y)}^{\beta(y)} f(x,y)\,dx\,dy$$

5.2 Exc #5 p 332

$$\int_0^2 \int_0^{y^2} y\,dx\,dy$$

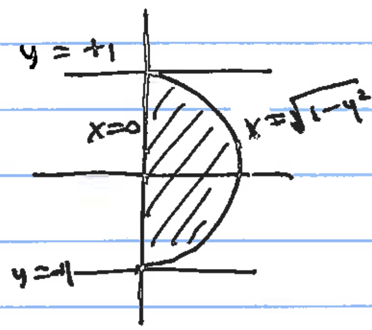
$0 \le y \le 2$
$0 \le x \le y^2$



$$= \int_0^2 \left(yx \Big|_{x=0}^{x=y^2} \right) dy = \int_0^2 (y^3 - 0)\,dy = \frac{y^4}{4} \Big|_{y=0}^{y=2} = 4$$

Exc #12

$$\int_{-1}^{1}\int_{0}^{\sqrt{1-y^2}} 3\,dx\,dy \qquad \begin{array}{l} -1 \le y \le 1 \\ 0 \le x \le \sqrt{1-y^2} \end{array}$$



Domain of integration

$$\int_{-1}^{1}\int_{0}^{\sqrt{1-y^2}} 3\,dx\,dy = \int_{-1}^{1} 3x\Big|_{x=0}^{x=\sqrt{1-y^2}} dy$$

$$= \int_{-1}^{1} 3\sqrt{1-y^2}\,dy = \int_{-\pi/2}^{\pi/2} 3\sqrt{1-\sin^2\theta}\cdot\cos\theta\,d\theta$$

$$y = \sin\theta$$

$$y=1 \quad \theta = \pi/2$$

$$y=-1 \quad \theta = -\pi/2$$

$$dy = \cos\theta\,d\theta$$

$$= \int_{-\pi/2}^{\pi/2} 3\cos^2\theta\,d\theta = \int_{-\pi/2}^{\pi/2} \frac{3}{2}(1+\cos 2\theta)\,d\theta$$

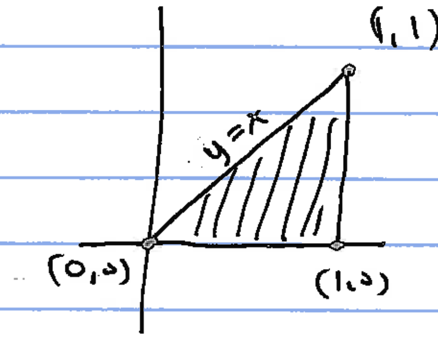
$$\cos^2\theta = \frac{1+\cos 2\theta}{2}$$

$$= \frac{3}{2}\left(\theta + \frac{\sin 2\theta}{2}\right)\Big|_{-\frac{\pi}{2}}^{\pi/2} = \frac{3}{2}\left[\left(\frac{\pi}{2} + \cancel{\frac{\sin\pi}{2}}^{\,0}\right) - \left(-\frac{\pi}{2} + \cancel{\frac{\sin -\pi}{2}}^{\,0}\right)\right] \boxed{= \frac{3\pi}{2}}$$

Prop $$\iint_D 1\, dA = \text{area } D$$ as long as LHS is calculable.

5.2 Exc #19

$$\iint_D e^{x^2}\, dA$$



Type I

$0 \le x \le 1$

$0 \le y \le x$

$$\int_0^1 \int_0^x e^{x^2}\, dy\, dx$$

Type II

$0 \le y \le 1$

$y \le x \le 1$

$$\int_0^1 \int_y^1 e^{x^2}\, dx\, dy$$

No for II : $\int e^{x^2} dx$ is not expressible in terms of finitely many simple functions

Yes for I $\displaystyle \int_0^1 \int_0^x e^{x^2}\,dy\,dx = \int_0^1 y e^{x^2} \Big|_{y=0}^{y=x} dx$

$$= \int_0^1 \left(x e^{x^2} - 0\right) dx = \int_0^1 \frac{1}{2} e^u\,du$$

$u = x^2$ $\quad$ $x=0 \quad u=0$

$du = 2x\,dx$ $\quad$ $x=1 \quad u=1$

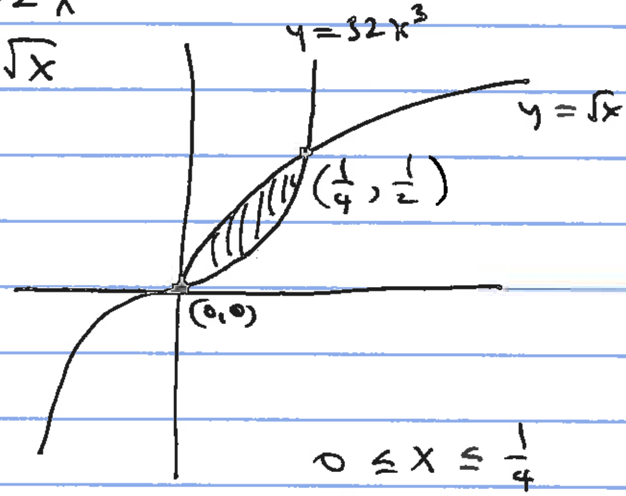
$$\text{I} = \frac{1}{2} e^u \Big|_{u=0}^{u=1} = \frac{1}{2}\left(e^1 - e^0\right) = \frac{1}{2}(e-1).$$

Exc # 16 p 333

$f = 3xy$

D bdd by $y = 32x^3$

$y = \sqrt{x}$



Pt of intersection

$32x^3 = \sqrt{x}$

$0 \le x \le \frac{1}{4}$

$32x^3 \le y \le \sqrt{x}$

$$x^{\frac{5}{2}} = \frac{x^3}{\sqrt{x}} = \frac{1}{32} = \frac{1}{2^5}$$

$$x^5 = \frac{1}{2^{10}}$$

$$x = \frac{1}{2^2} = \frac{1}{4} \implies y = \sqrt{\tfrac{1}{4}} = \frac{1}{2}$$

$$\text{Integral} = \int_0^{1/4}\int_{32x^3}^{\sqrt{x}} 3xy\,dy\,dx = \int_0^{1/4} \frac{3xy^2}{2}\Bigg|_{32x^3}^{\sqrt{x}} dx$$

$$= \int_0^{1/4} \frac{3x^2}{2} - \frac{3x(32x^3)^2}{2}\,dx$$

$$= \int_0^{1/4}\left(\frac{3}{2}x^2 - 3\cdot 512\,x^7\right)dx = \frac{x^3}{2} - 3\cdot 64\,x^8\Bigg|_0^{\frac{1}{4}}$$

$$= \left(\frac{1}{4}\right)^3 \cdot \frac{1}{2} - 3 \cdot 64 \left(\frac{1}{4}\right)^8 = \frac{1}{2^7} - \frac{3}{2^{10}} = \frac{1}{2^{10}}(8-3)$$

$64 = 2^6$ $\quad 4^8 = 2^{16}$

$$= \frac{5}{2^{10}} = \frac{5}{1024}$$