March 24, 2017

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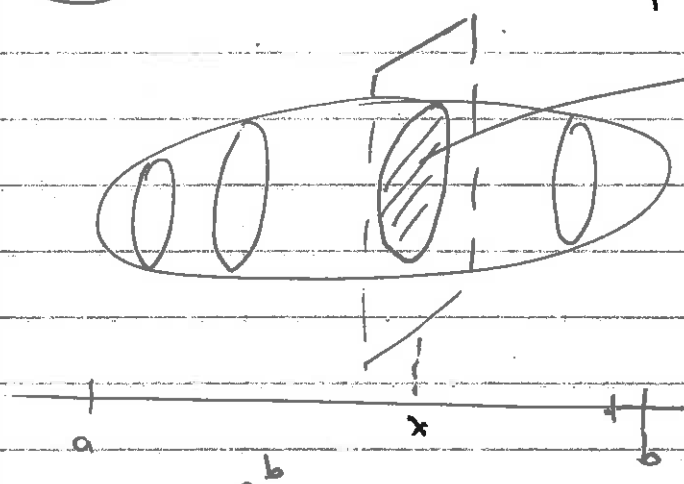
Chap 5

5.1 Cavalieri's Principle (Calc I)



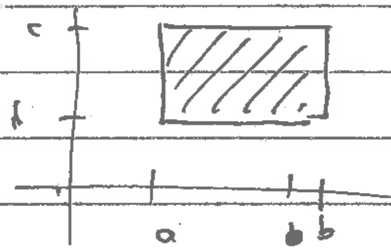
Area = $A(x)$ of the slice

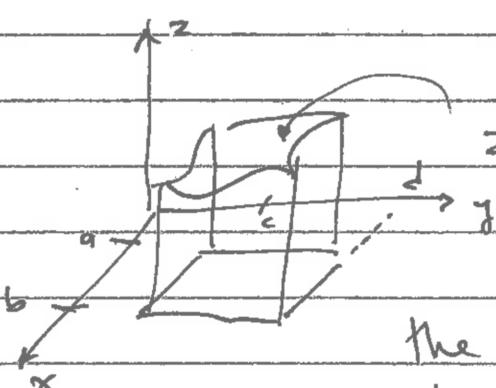
- Sliced by parallel planes
- Want Volume of the solid

$$\int_a^b A(x)\,dx = \text{Volume}.$$

$f: [a,b]\times[c,d] \longrightarrow \mathbb{R}$

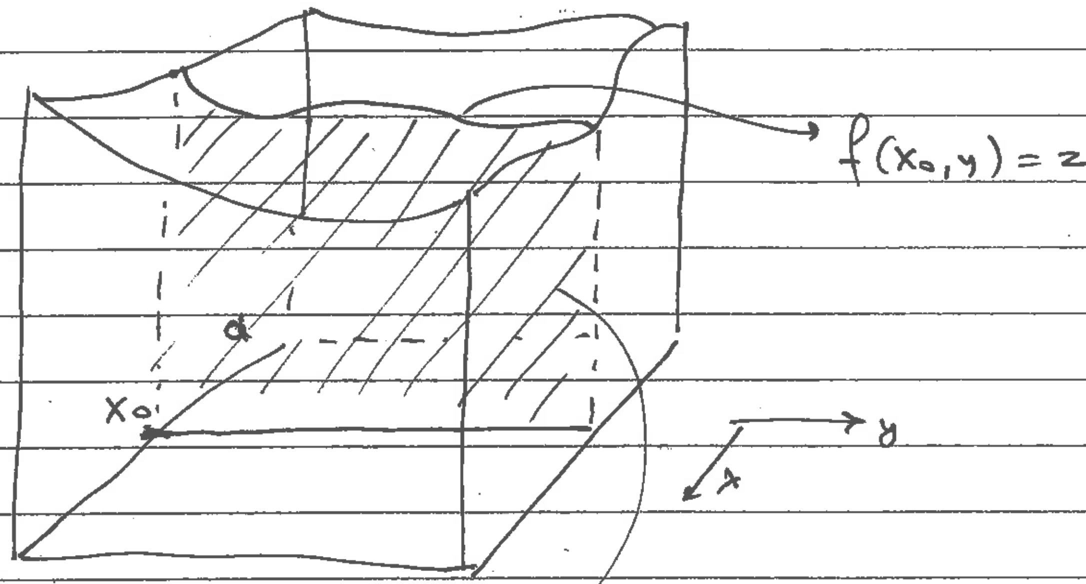
$\{(x,y) \mid a \le x \le b,\ c \le y \le d\}$





$z = f(x,y) \geq 0$

Want the volume V of the solid region bounded below by xy-plane; by the graph of $z = f(x,y)$ on top, and by the vertical planes $x = a$, $x = b$, $y = c$, $y = d$ } on the 4 sides.



area of the slice } $A(x_0) = \int_c^d f(x_0, y)\, dy$

<u>Thm:</u>

If f is continuous on $[a,b] \times [c,d]$

$$V = \int_a^b \left(\int_c^d f(x,y)\, dy \right) dx$$

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Iterated Integrals

$$\int_c^d \boxed{\int_a^b f(x,y)\,dx}\; dy$$

1. integrate $f(x,y)$ wrt x, keeping all y's constant
$\underbrace{\text{Substitute } x=a,\ x=b \ldots}$

You'll get a ~~function~~ function of y : $A(y)$

2. $\displaystyle\int_c^d A(y)\,dy \longrightarrow$ finish it.

$$\int_c^d \left(\int_a^b f(x,y)\,dx\right) dy$$

x is first
y is second

$$\int_a^b \left(\int_c^d f(x,y)\,dy\right) dx$$

y is first
x is second.

different notations used by many people.

$$\int_c^d dy \int_a^b f(x,y)\,dx$$

x is first
y is second

$$\int_a^b dx \int_c^d f(x,y)\,dy$$

y is first
x is second.

5.1 Exc #2 p 313

$$\int_0^{\pi} \int_1^{2} y \sin x \, dy \, dx$$

$$= \int_0^{\pi} \left(\frac{1}{2} y^2 \sin x \Big|_{y=1}^{y=2} \right) dx$$

$$= \int_0^{\pi} \left(2 \sin x - \frac{1}{2} \sin x \right) dx$$

$$= \int_0^{\pi} \frac{3}{2} \sin x \, dx$$

$$= -\frac{3}{2} \cos x \Big|_{x=0}^{x=\pi}$$

$$= \left(-\frac{3}{2} \underbrace{\cos \pi}_{-1} \right) - \left(-\frac{3}{2} \cos 0 \right)$$

$$= \frac{3}{2} + \frac{3}{2} = 3$$

Ex #6 p 313

$$\int_1^9 \int_1^e \frac{\ln\sqrt{x}}{xy}\,dx\,dy$$

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$$\boxed{\begin{aligned} &\int \frac{\ln\sqrt{x}}{x}\,dx = \int \frac{1}{2}\frac{\ln x}{x}\,dx \qquad u = \ln x \\ &= \int \frac{1}{2}u\,du = \frac{u^2}{4} + C \qquad du = \frac{1}{x}dx \\ &\qquad = \frac{(\ln x)^2}{4} + C \end{aligned}}$$

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$$\int_1^9 \int_1^e \frac{\ln\sqrt{x}}{xy}\,dx = \int_1^9\int_1^e \frac{1}{y}\frac{1}{2}\frac{\ln x}{x}\,dx\,dy$$

$$= \int_1^9 \frac{1}{y}\cdot\frac{(\ln x)^2}{4}\Bigg|_{x=1}^{x=e} dy$$

$$= \int_1^9 \frac{1}{y}\Bigg(\underbrace{\frac{(\ln e)^2}{4} - \frac{(\ln 1)^2}{4}}_{\frac{1}{4}}\Bigg)\,dy$$

$$= \int_1^9 \frac{1}{4y}\,dy = \frac{1}{4}\ln|y|\Bigg|_{y=1}^{y=9}$$

$$= \frac{1}{4}\left(\ln 9 - \cancelto{0}{\ln 1}\right) = \frac{\ln 9}{4}.$$