4.1 Higher order Taylor polynomials:

Def $f: \underbrace{X}_{\text{open}} \subseteq \mathbb{R}^n \longrightarrow \mathbb{R}^1$, $a \in X$

f is at least m times diffble (continuously) at a.

$$P_0(x;a) = f(a)$$

$$P_1(x;a) = f(a) + \sum_{k=1}^{n} \frac{\partial f}{\partial x_k}(a)(x_k - a_k)$$

$$P_2(x;a) = P_1(x;a) + \frac{1}{2} \sum_{i,j}^{n} \frac{\partial^2 f}{\partial x_j \partial x_i}(a)(x_i - a_i)(x_j - a_j)$$

$1 \le k \le m$ $\quad P_k(x;a) = P_{k-1}(x;a) +$

$$+ \frac{1}{k!} \sum_{i_1 i_2 \cdots i_k} \frac{\partial^k f}{\partial x_{i_k} \partial x_{i_{k-1}} \cdots \partial x_{i_2} \partial x_{i_1}}(a)\,(x_{i_1} - a_{i_1})(x_{i_2} - a_{i_2}) \cdots (x_{i_k} - a_{i_k})$$

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Exc #10 $f = e^{2x+y}$ $a = (0,0)$

Find P_1, P_2, P_3 of f at a.

$f(0,0) = 1$

$f_x = \frac{\partial f}{\partial x} = 2e^{2x+y}$ $\frac{\partial f}{\partial x}(0,0) = 2$

$f_y = \frac{\partial f}{\partial y} = e^{2x+y}$ $\frac{\partial f}{\partial y}(0,0) = 1.$

$P_1 = 1 + 2(x-0) + 1(y-0)$

$f_{xx} = 4e^{2x+y}$ $f_{xx}(0,0) = 4$

$f_{yx} = f_{xy} = 2e^{2x+y}$ $f_{yx}(0,0) = f_{xy}(0,0) = 2$

$f_{yy} = 1 \cdot e^{2x+y}$ $f_{yy}(0,0) = 1$

$P_2 = 1 + 2(x-0) + 1(y-0) + \cdots$

$\cdots \frac{1}{2}\left[4(x-0)^2 + 4(x-0)(y-0) + 1(y-0)^2\right].$

(PTO) for P_3

at $(0,0)$

$f_{xxx} = 8e^{2x+y}$ — 8

$f_{xxy} = 4e^{2x+y}$ — 4
$f_{xyx} =$ "
$f_{yxx} =$ "

$f_{xyy} = 2e^{2x+y}$ — 2
$f_{yxy} =$ "
$f_{yyx} =$ "

$f_{yyy} = e^{2x+y}$ — 1

$$P_3 = 1 + 2(x-0) + 1(y-0) + \frac{1}{2}\left[4(x-0)^2 + 4(x-0)(y-0) + 1(y-0)^2\right] +$$

$$\therefore + \frac{1}{6}\left[8(x-0)^3 + 12(x-0)^2(y-0) + 6(x-0)(y-0)^2 + 1(y-0)^3\right]$$

Ex

$f = x^3 + 6xyz + xy^2 - 7z$

Want p_2, at $(1,2,0)$

	at $(1,2,0)$	
$f = x^3 + 6xyz + xy^2 - 7z$	5	
$f_x = 3x^2 + 6yz + y^2$	7	First order
$f_y = 6xz + 2xy$	4	
$f_z = 6xy - 7$	5	
$f_{xx} = 6x$	6	
$f_{xy} = 6z + 2y$	4	
$f_{yy} = 2x$	2	2nd order
$f_{xz} = 6y$	12	
$f_{yz} = 6x$	6	
$f_{zz} = 0$	0	

Ex $f = x^3 + 6xyz + xy^2 - 7z$

Want P_1, P_2 at $(1,2,0)$

$$P_2 = 5 + \begin{bmatrix} 7 & 4 & 5 \end{bmatrix} \begin{bmatrix} x-1 \\ y-2 \\ z-0 \end{bmatrix} + \frac{1}{2} \begin{bmatrix} x-1 & y-2 & z-0 \end{bmatrix} \begin{bmatrix} 6 & 4 & 12 \\ 4 & 2 & 6 \\ 12 & 6 & 0 \end{bmatrix} \begin{bmatrix} x-1 \\ y-2 \\ z-0 \end{bmatrix}$$

$$= 5 + 7(x-1) + 4(y-2) + 5(z-0) + \cdots$$

$$\cdots + \frac{1}{2}\Big(6(x-1)^2 + 2(y-2)^2 + 0(z-0)^2 + \cdots$$

$$\cdots + 8(x-1)(y-2) + 24(x-1)(z-0) + 12(y-2)(z-0) \Big)$$

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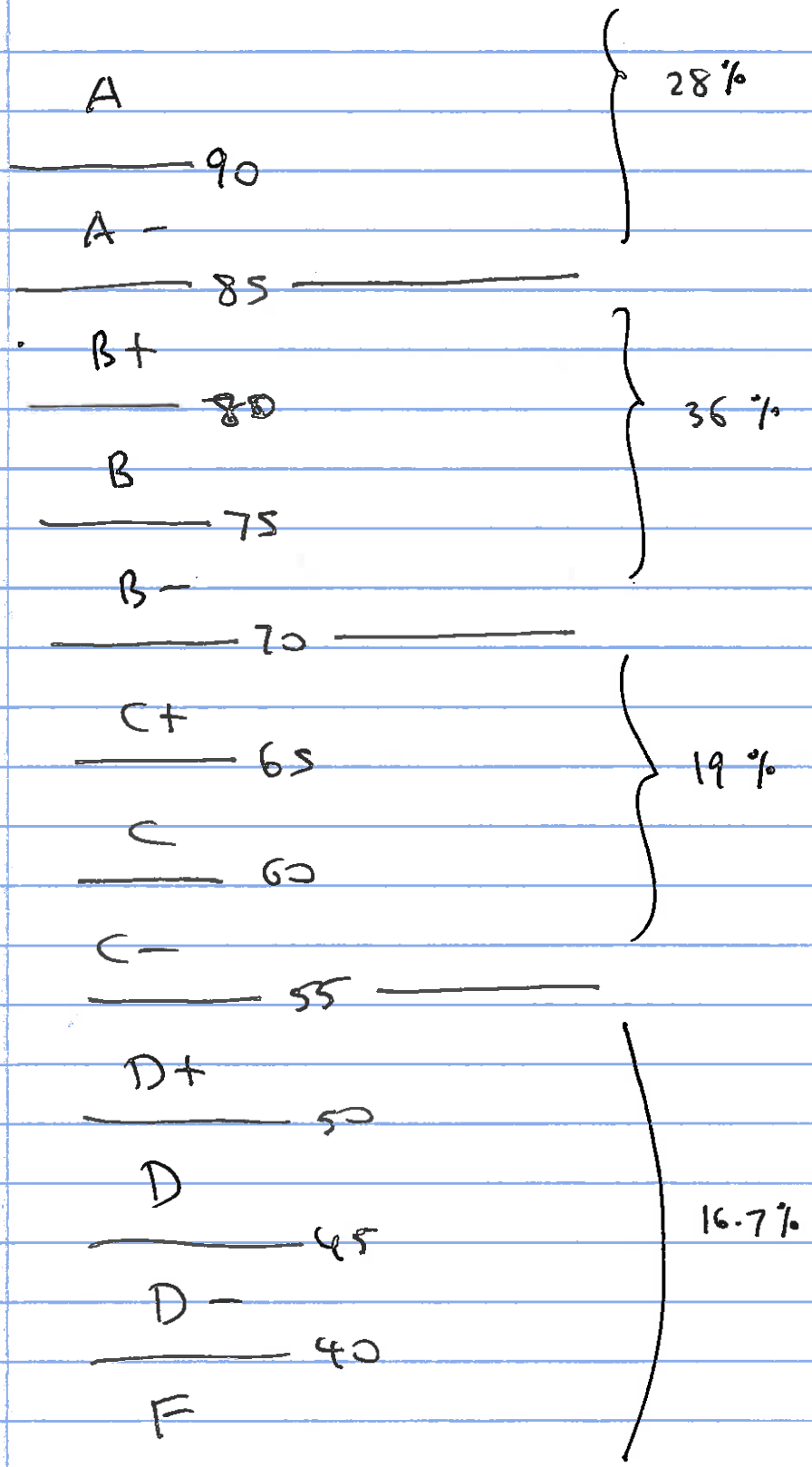
Sections 111
MT 1 Exam only



Ave 73.2
median 79