March 2, 2017



3.1 Continue

Ex #16

$$\vec{x}(t) = 4\cos t\, i + 3\sin t\, j + 5t\, k$$

want tangent line to this curve when $t = \pi/3$

$$\vec{x}'(t) = -4\sin t\, i - 3\cos t\, j + 5k$$

$$\vec{x}\left(\tfrac{\pi}{3}\right) = 4\cdot\tfrac{1}{2} i - 3\cdot\tfrac{\sqrt{3}}{2} j + 5\cdot\tfrac{\pi}{3} k$$

$$\vec{x}'\left(\tfrac{\pi}{3}\right) = -4\cdot\tfrac{\sqrt{3}}{2} i - 3\cdot\tfrac{1}{2} j + 5k$$

$$r(t) = \left(2, \tfrac{-3\sqrt{3}}{2}, \tfrac{5\pi}{3}\right) + t\left(-2\sqrt{3}, -\tfrac{3}{2}, 5\right)$$

or

$$r(t) \quad \left(2, \tfrac{-3\sqrt{3}}{2}, \tfrac{5\pi}{3}\right) + \left(t - \tfrac{\pi}{3}\right)\left(-2\sqrt{3}, -\tfrac{3}{2}, 5\right)$$

Ex Parametrize the curve of intersection of the parabolic cylinder $z = y^2$ and the circular

cylinder $x^2 + y^2 = 4$

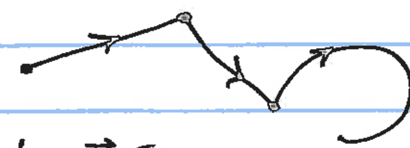
$$\Big(\underbrace{\overbrace{2\cos t}^{x}, \overbrace{2\sin t}^{y}}_{\text{circle of radius 2}}, \underbrace{4\sin^2 t}_{z=y^2}\Big)$$

3.2

## Lengths of curves

**Def** Let $\vec{x} : [a,b] \to \mathbb{R}^n$ be a piecewise $C^1$ parametrized curve

finitely many corners



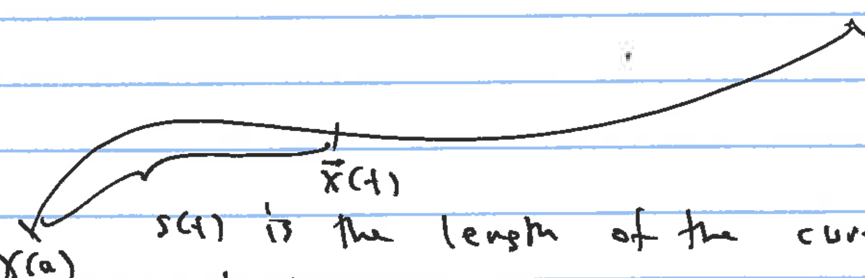
We define the length of $\vec{x}(t)$

$$L(\vec{x}) = \int_a^b \|\vec{x}'(t)\|\,dt$$

Caution! if $x(t)$ is not 1-1, then some parts of the path will be counted for more than once.

**Defn** $s(t) = \int_a^t \|\vec{x}'(u)\|\,du$

arclength function




$s(t)$ is the length of the curve between $\vec{x}(a)$ & $\vec{x}(t)$

Ex 1

$$\vec{x}(t) = (\cos 3t,\ \sin 3t,\ 4t)$$

$$0 \le t \le 2\pi$$

Want length of $\vec{x}(t)$ on $[0, 2\pi]$.

$$\vec{x}'(t) = (-3\sin 3t,\ 3\cos 3t,\ 4)$$

$$\|\vec{x}'(t)\| = \sqrt{9\sin^2 3t + 9\cos^2 3t + 16}$$
$$= 5$$

$$\text{length} = \int_0^{2\pi} \|\vec{x}'(t)\|\,dt = \int_0^{2\pi} 5\cdot dt = 10\pi$$

Ex 2 $\quad \gamma(t) = \left(\ln t,\ \frac{t^2}{2},\ \sqrt{2}\,t\right) \qquad 1 \le t \le 4.$

Find length & arclength function

$$\gamma'(t) = \left(\frac{1}{t},\ t,\ \sqrt{2}\right)$$

$$\|\gamma'(t)\| = \sqrt{\frac{1}{t^2} + t^2 + 2} = t + \frac{1}{t}$$

$$s(t) = \int_1^t \left(u + \frac{1}{u}\right) du = \frac{u^2}{2} + \ln u \Big|_1^t =$$

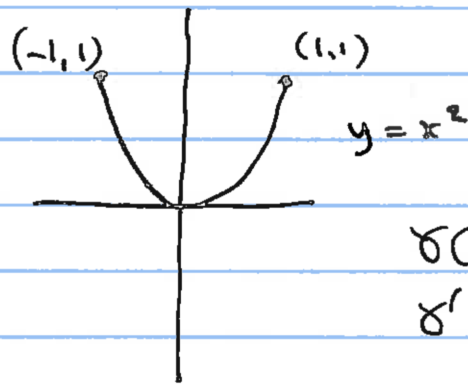
$$= \left(\frac{t^2}{2} + \ln t\right) - \left(\frac{1}{2} + \ln 1\right) = \frac{t^2}{2} + \ln t - \frac{1}{2}.$$

length of the curve on $[1,4]$

$$s(4) = \int_1^4 \left(u + \frac{1}{u}\right) du = 8 + \ln 4 - \frac{1}{2}.$$

Caution: The exact values of the many of the length integrals are not easy to find; most of the time, they are impossible to find.

Ex (a)

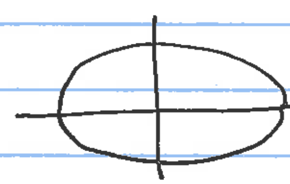



$$\gamma(t) = (t, t^2)$$
$$\gamma'(t) = (1, 2t)$$

$$\text{length} = \int_{-1}^{1} \sqrt{1+4t^2}\, dt$$

Doable! Not Easy
HW to do it.

(b)
$$\gamma(t) = (2\cos t, 3\sin t)$$
$$\gamma'(t) = (-2\sin t, 3\cos t)$$



$$\text{length} = \int_0^{2\pi} \underbrace{\sqrt{4\sin^2 t + 9\cos^2 t}\, dt}_{\text{not expressible in terms of finitely many simple functions}}$$