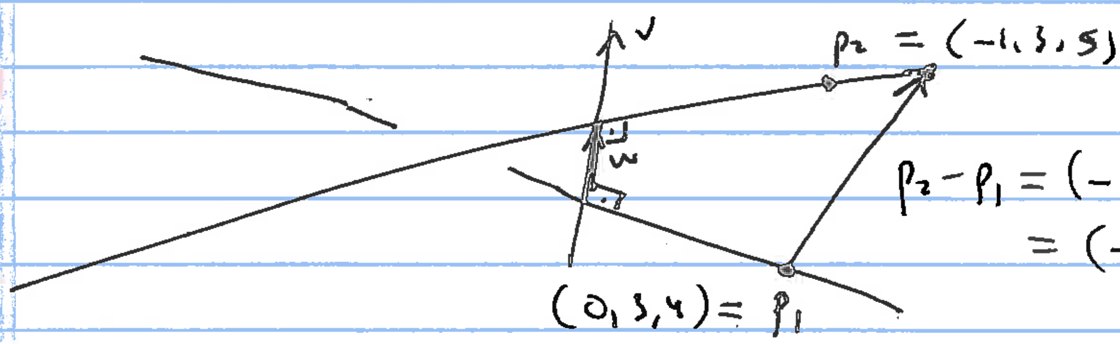
March 2, 2017

Review ①

p48 1.5 # 27 Exc.

$$l_1(t) = t(8,-1,0) + (-1,3,5)$$
$$l_2(t) = t(0,3,1) + (0,3,4)$$



$$p_2 - p_1 = (-1,3,5) - (0,3,4) = (-1,0,1)$$

$$v \parallel (8,-1,0) \times (0,3,1) = \begin{vmatrix} i & j & k \\ 8 & -1 & 0 \\ 0 & 3 & 1 \end{vmatrix}$$

$$= (-1,-8,24)$$

$$w = \operatorname{proj}_v (p_2 - p_1)$$

$$= \operatorname{proj}_{(-1,-8,24)} (-1,0,1) = \frac{(-1,0,1)\cdot(-1,-8,24)}{(-1,-8,24)\cdot(-1,-8,24)}(-1,-8,24)$$

$$= \frac{1+0+24}{\underbrace{1+64+576}_{641}}(-1,-8,24)$$

$$\|(-1,8,24)\| = \sqrt{641}$$

$$\|w\| = \frac{25}{\sqrt{641}} = \left\| \frac{25}{641}(-1,-8,24) \right\| = \left|\frac{25}{641}\right| \|(-1,-8,24)\|$$
$$= \frac{25}{641}\sqrt{641}$$

$$\begin{array}{r} 24 \\ \times 24 \\ \hline 96 \\ 48 \\ \hline 576 \\ 64 \\ 1 \\ \hline 641 \end{array}$$

Practice Test #8

$x=3,\ y=0.$

$g = xe^{-y}$

$g(3,0) = 3$

$\frac{\partial g}{\partial x}(3,0) = 1$

$\frac{\partial g}{\partial y}(3,0) = -3$

(a) $\frac{\partial g}{\partial x} = e^{-y}$

$\frac{\partial g}{\partial y} = -xe^{-y}$

$$\nabla f = (e^{-y}, -xe^{-y})$$

$$Df = [\, e^{-y} \quad -xe^{-y} \,]$$

(b)

$$z = g(a,b) + \frac{\partial g}{\partial x}(a,b)(x-a) + \frac{\partial g}{\partial y}(a,b)(y-b)$$

$$z = 3 + 1\cdot(x-3) - 3(y-0)$$

$$z = \cancel{3} + x - \cancel{3} - 3y$$

$$\boxed{z = x - 3y}$$ Tangent plane

$h(x,y) = x - 3y$ Tangent plane approximation

(c)

$$g(3.1, 0.3) \cong 3 + (3.1-3) - 3(0.3-0)$$

$$= 3 + 0.1 - 0.9$$

$$= 2.2$$

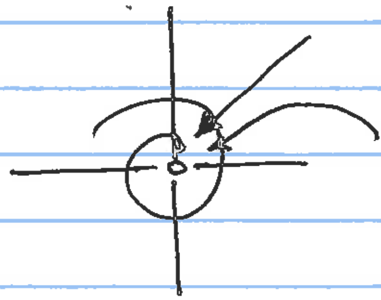
(d) $\nabla f(3,0) = (1,-3) \longrightarrow$ direction fastest increase $\frac{(1,-3)}{\sqrt{10}}$

steepest rate of increase $\sqrt{10}$

Practice

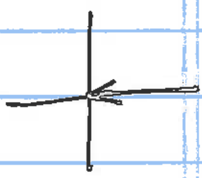
10 b (ii)

$$\lim_{(x,y)\to(0,0)} \frac{x^2}{x^2+y^2} \quad \underline{\underline{\text{DNE}}}$$



$(x,0) \to (0,0)$

$(0,y) \to (0,0)$



$$\lim_{(x,0)\to(0,0)} \frac{x^2}{x^2+0} = \lim_{x\to 0} 1 = 1$$

$\neq$

$$\lim_{(0,y)\to(0,0)} \frac{0}{0+y^2} = \lim_{y\to 0} 0 = 0$$

$f_x \qquad \dfrac{\partial f}{\partial x} \qquad$ acceptable notations.

2.1 #18
p96

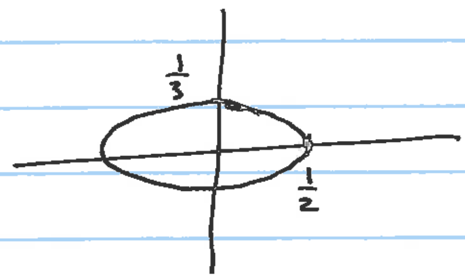
$f(x,y) = 4x^2 + 9y^2$.

$c = -1$ $\quad 4x^2 + 9y^2 = -1$ $\quad$ no sol'n

$c = 0$ $\quad 4x^2 + 9y^2 = 0$ $\quad (x,y) = (0,0)$

$c = 1$ $\quad 4x^2 + 9y^2 = 1$ $\quad \dfrac{x^2}{(1/2)^2} + \dfrac{y^2}{(1/3)^2} = 1$

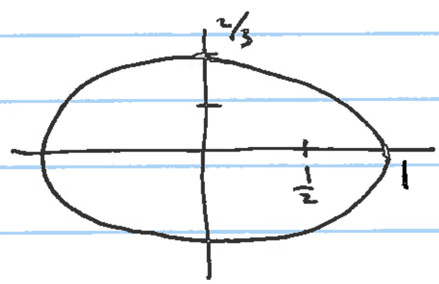
$c = 4$

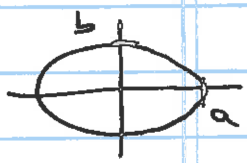


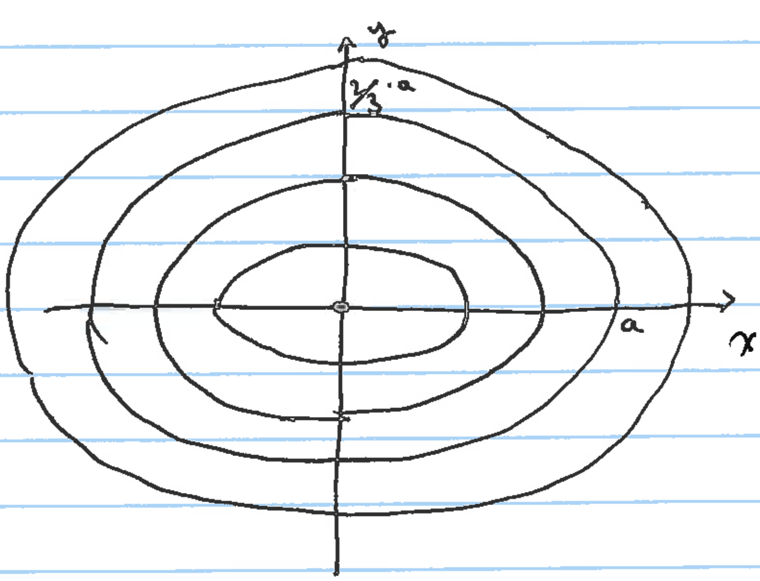
$4x^2 + 9y^2 = 4$

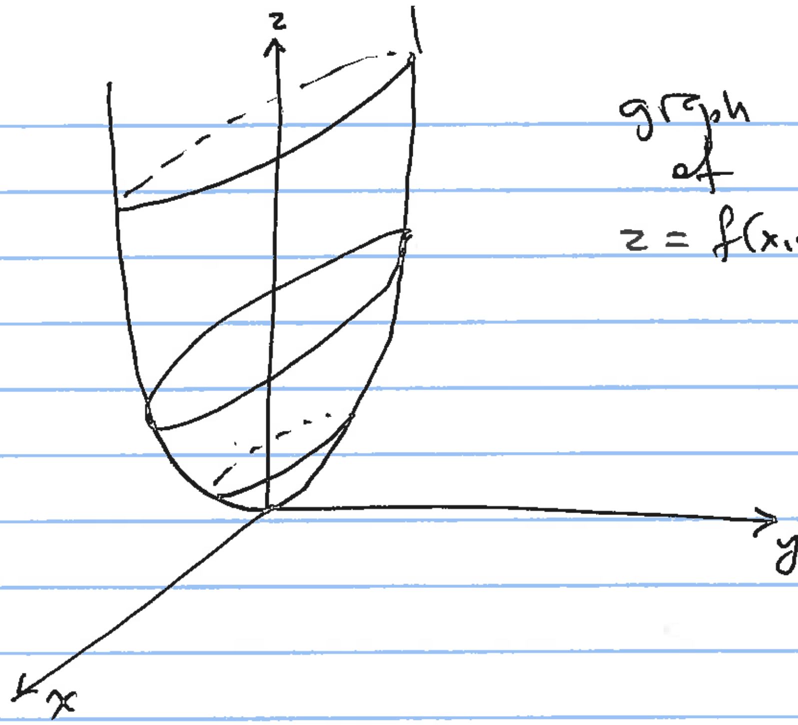
$x^2 + \frac{9}{4}y^2 = 1$

$\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ $\quad \dfrac{x^2}{1^2} + \dfrac{y^2}{(\frac{2}{3})^2} = 1$





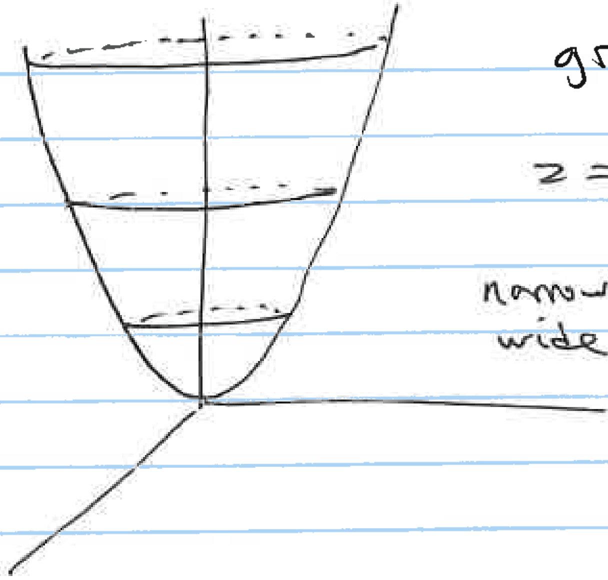




graph of

$z = f(x,y) = 4x^2 + 9y^2$

Ex.



graph of $z = 4x^2 + y^2$

narrow in x direction
wide in y direction

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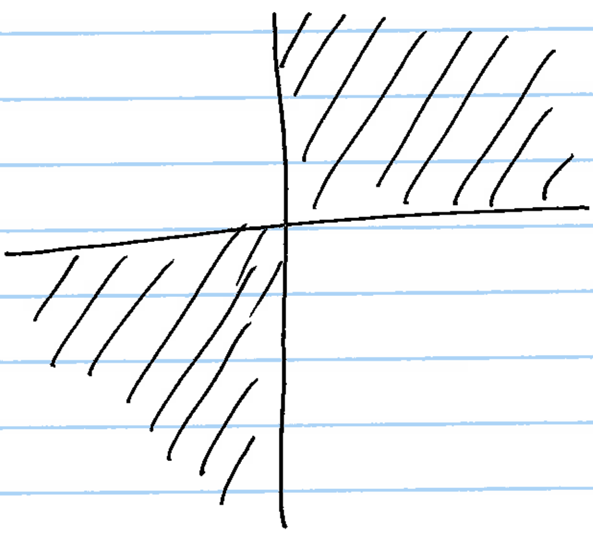
p183 #3

$f(x,y) = \sqrt{xy}$: $\{(x,y) \mid xy \geq 0\} = \{(x,y) \mid \begin{pmatrix} x \geq 0 \\ y \geq 0 \end{pmatrix} \text{ or } \begin{pmatrix} x \leq 0 \\ y \leq 0 \end{pmatrix}\}$



Domain
<u>is closed</u>

since the boundary
(= union of x & y axes)
$\subseteq$ Domain

<u>Range</u> $:= [0, \infty)$

$\sqrt{} \geq 0$, no negative values in the range.

$f(4,4) = 4$

$\forall c \geq 0 \quad f(c,c) = c$. So, all $c \geq 0$, $c \in$ Range.

2.5 #3

$$t \longrightarrow \left.\begin{matrix} x \\ y \\ z \end{matrix}\right\} \xrightarrow{P} w$$

$x = 2\cos t$ $\quad w = P(x,y,z) = \dfrac{6x^2 z}{y}$ atm.

$y = 2\sin t$

$z = 3t$

$$\frac{d}{dt} P(x(t), y(t), z(t))$$

$$= \frac{\partial P}{\partial x} \cdot \frac{\partial x}{\partial t} + \frac{\partial P}{\partial y} \cdot \frac{\partial y}{\partial t} + \frac{\partial P}{\partial z} \frac{\partial z}{\partial t}$$

$$t = \pi/4 \longmapsto \left.\begin{matrix} x = \frac{\sqrt{2}}{2} = \sqrt{2} \\ y = \sqrt{2} \\ z = \frac{3\pi}{4}. \end{matrix}\right\}$$

$$\frac{d}{dt} P(x,y,z)\Big|_{t=\pi/4} = \frac{\partial P}{\partial x}\Big|_{(\sqrt{2}, \sqrt{2}, \frac{3\pi}{4})} \cdot \frac{\partial x}{\partial t}\Big|_{t=\pi/4} +$$

$$\frac{\partial P}{\partial y}\Big|_{(\sqrt{2}, \sqrt{2}, \frac{3\pi}{4})} \cdot \frac{\partial y}{\partial t}\Big|_{t=\pi/4} +$$

$$\frac{\partial P}{\partial z}(\sqrt{2}, \sqrt{2}, \tfrac{3\pi}{4}) \cdot \frac{\partial z}{\partial t}\Big|_{t=\pi/4}.$$

$\dfrac{\partial x}{\partial t} = -2\sin t$ at $\pi/4$ $\quad \dfrac{\partial x}{\partial t}\Big|_{t=\pi/4} = -\sqrt{2}$

$\dfrac{\partial y}{\partial t} = 2\cos t$ $\quad \dfrac{\partial y}{\partial t}\Big|_{t=\pi/4} = \sqrt{2}$

$\dfrac{\partial z}{\partial t} = 3$ $\quad \dfrac{\partial z}{\partial t}\Big|_{t=\pi/4} = 3.$

$$P = \frac{6x^2 z}{y} \qquad \text{at } \left(\sqrt{2}, \sqrt{2}, \frac{3\pi}{4}\right)$$

$$\frac{\partial P}{\partial x} = \frac{12xz}{y} \qquad \frac{12 \cdot \cancel{\sqrt{2}} \cdot \frac{3\pi}{4}}{\cancel{\sqrt{2}}} = 9\pi$$

$$\frac{\partial P}{\partial y} = -\frac{6x^2 z}{y^2} \qquad -\frac{6 \cdot \cancel{2} \cdot \frac{3\pi}{4}}{\cancel{2}} = -\frac{9}{2}\pi$$

$$\frac{\partial P}{\partial z} = \frac{6x^2}{y} \qquad \frac{6 \cdot 2}{\sqrt{2}} = 6\sqrt{2}$$

$$\frac{d}{dt} P(x,y,z) \Big|_{t=\frac{\pi}{4}} = 9\pi \cdot (-\sqrt{2}) + \frac{-9}{2}\pi \cdot \sqrt{2} + 6\sqrt{2} \cdot 3$$

$P : \mathbb{R}^3 \to \mathbb{R}^1$ $\qquad P(x(t), y(t), z(t))$ $\qquad t \xrightarrow{f} (x,y,z) = f(t)$

$\qquad \mathbb{R} \longrightarrow \mathbb{R}^3$

$$\frac{d}{dt} P(f(t))$$

$$D(P \circ f)\left(\frac{\pi}{4}\right) = \underset{1 \times 3}{DP\left(f\left(\frac{\pi}{4}\right)\right)} \cdot \underset{3 \times 1}{Df\left(\frac{\pi}{4}\right)}$$

$$= \begin{bmatrix} 9\pi & -\frac{9\pi}{2} & 6\sqrt{2} \end{bmatrix} \begin{bmatrix} -\sqrt{2} \\ \sqrt{2} \\ 3 \end{bmatrix}$$

$$= \left[-9\pi\sqrt{2} - \frac{9}{2}\pi\sqrt{2} + 18\sqrt{2} \right]$$

Simplify: $-9\pi\sqrt{2} - \frac{9}{2}\pi\sqrt{2} + 18\sqrt{2} =$

$= \sqrt{2}\left(-9\pi - \frac{9}{2}\pi + 18\right) = \sqrt{2}\left(-\frac{27\pi}{2} + \frac{36}{2}\right)$

$= \frac{\sqrt{2}}{2}(36 - 27\pi) = \frac{36 - 27\pi}{\sqrt{2}}$