Chap III.

3.1 Defn A continuous function

$$\vec{x} : [a,b] \subseteq \mathbb{R}^1 \longrightarrow \mathbb{R}^n \text{ is called a path}$$

or a parametrized curve.

$\vec{x}(a)$, $\vec{x}(b)$ are called the end points.

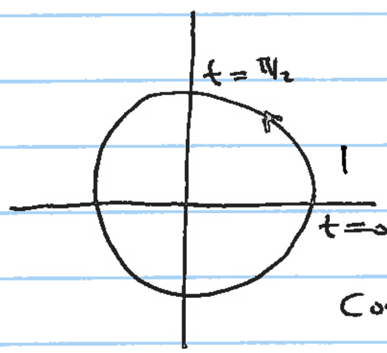
Ex . $\vec{x}(t) = (3+t,\ 5-t,\ 6t+11) : [0,1] \longrightarrow \mathbb{R}^3$

parametrized line segment with end pts

$(3,5,11)$, $(4,4,17)$.

Ex . $\vec{x}(t) = (\cos t, \sin t)$

$t \in \mathbb{R}$.



$x^2+y^2=1$

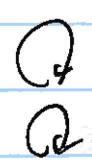
Counter clockwise

$t=0 \quad \vec{x}(0) = (1,0)$

$t=\pi/2 \quad \vec{x}(\frac{\pi}{2}) = (0,1)$

Clockwise? $(\sin t, \cos t)$

$(\cos t, -\sin t)$

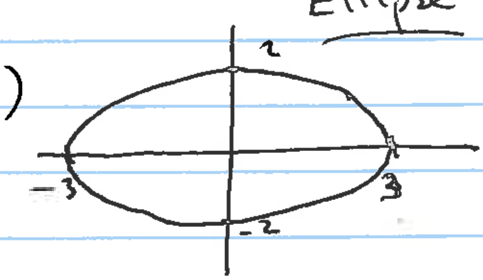


Ex . $\vec{x}(t) = (\underbrace{3\cos t}_{x}, \underbrace{2\sin t}_{y})$

$\vec{x}(0) = (3,0)$

$\vec{x}(\frac{\pi}{2}) = (0,2)$

Ellipse



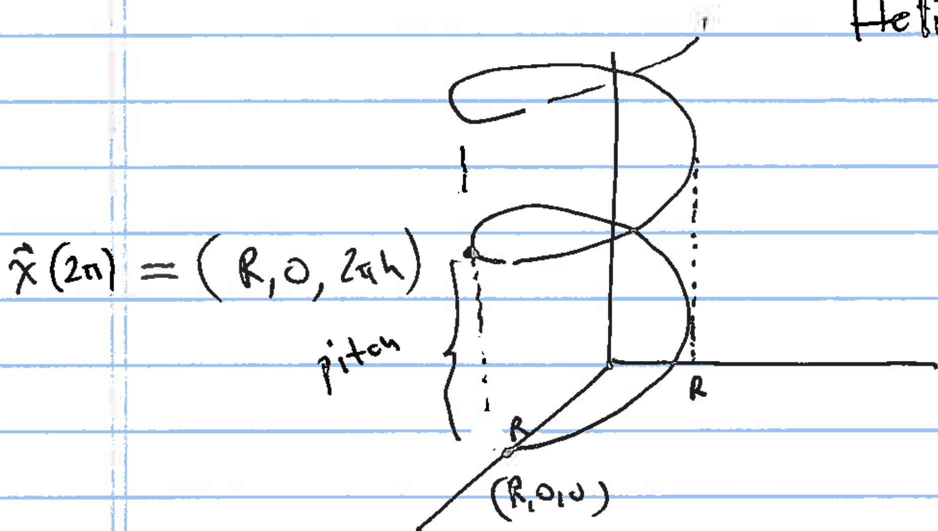
$x = 3\cos t$
$y = 2\sin t$

$$1 = \cos^2 t + \sin^2 t = \left(\frac{x}{3}\right)^2 + \left(\frac{y}{2}\right)^2 = \frac{x^2}{9} + \frac{y^2}{4}$$

Ex. $\vec{x}(t) = (R\cos t, R\sin t, ht)$

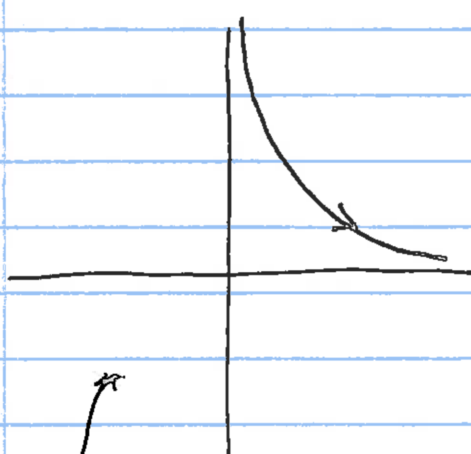
$R, h > 0$

Helix



Exc #2 p200 $\vec{x}(t) = e^t i + e^{-t} j = (e^t, e^{-t}) = (x, y)$

$0 < x = e^t$
$0 < y = e^{-t}$ } $xy = 1$



$t \uparrow \quad x \uparrow$
$t \uparrow \quad y \downarrow$

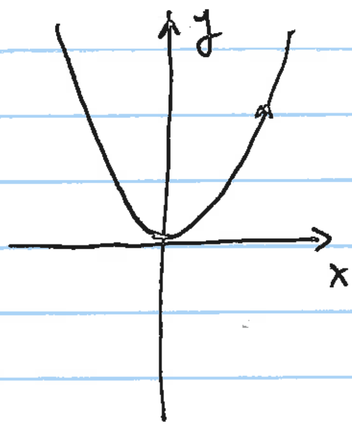
since $x>0$, $y>0$, $\vec{x}$ doesn't trace anything in the III rd quadrant.

p200

Exc # 6 $\vec{x}(t) = (t, t^2, t^3)$

Top view from z-axis direction

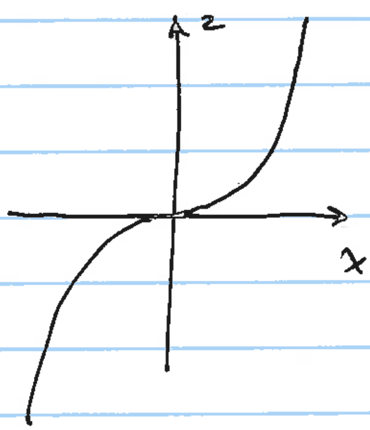
$(t, t^2) = (x, y)$



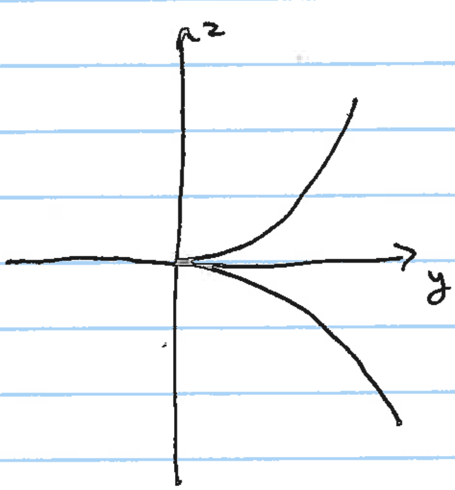
$x = t$, $y = t^2$ $\Big\}$ $y = x^2$

Side view from y-axis direction

$(x, z) = (t, t^3)$



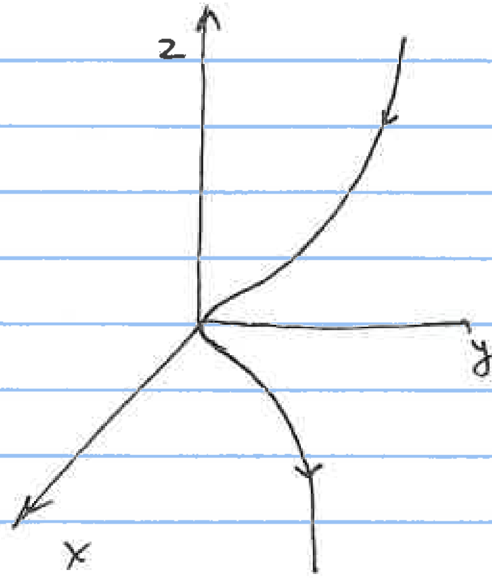
side view from x-axis direction



$(y, z) = (t^2, t^3)$

$y = t^2$, $z = t^3$ $\Big\}$ $y^3 = z^2$

cusp.



3-d picture of (t, t^2, t^3)

Defn Let $\vec{x} : I \subseteq \mathbb{R}^1 \longrightarrow \mathbb{R}^n$ be a diff'ble parametrized curve. Define

Velocity $\quad v(t) = \dfrac{d\vec{x}}{dt} = \dot{\vec{x}}(t) = \vec{x}\,'(t)$

Speed $\quad \|v(t)\| = \|x'(t)\|$

Acceleration $\quad v'(t) = \dfrac{d^2\vec{x}}{dt^2} = \|\vec{x}\,''(t)\|.$

Unit tangent vector $= \dfrac{\vec{x}\,'(t)}{\|x'(t)\|}$ direction of the velocity if $\neq 0$.

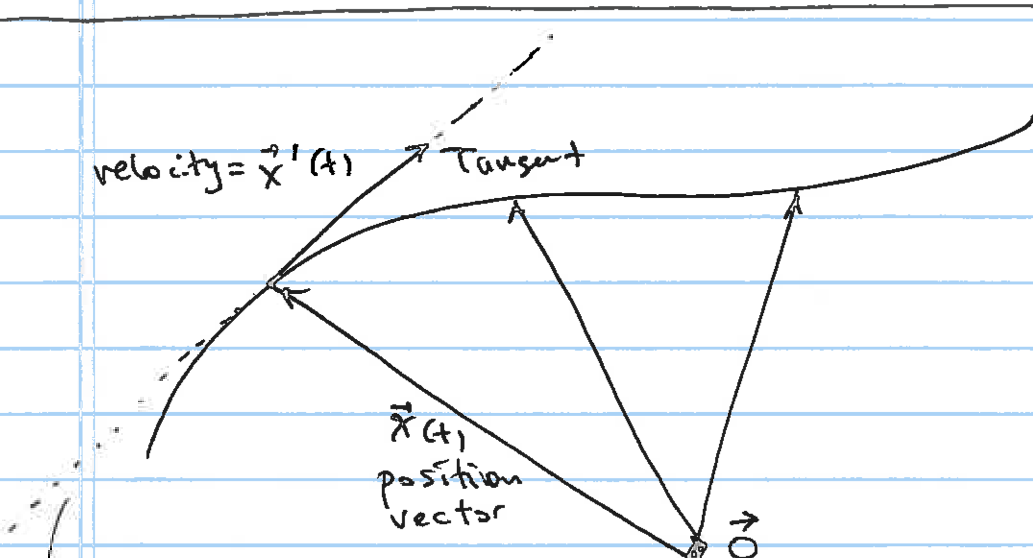
Ex: Sec #8 p200

$$\vec{x}(t) = 5\cos t\, i + 3\sin t\, j$$

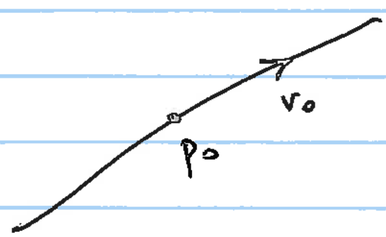
velocity $\vec{x}'(t) = -5\sin t\, i + 3\cos t\, j$

acceleration $\vec{x}''(t) = -5\cos t\, i - 3\sin t\, j$

speed $= \|\vec{x}'(t)\| = \sqrt{25\sin^2 t + 9\cos^2 t}$



recall $r(t) = p_0 + t v_0$
parametric line



$$\vec{r}(t) = \vec{x}(t_0) + t \cdot \vec{x}'(t_0)$$

eq of the tangent line to $x(t)$ at $x(t_0)$, if $\vec{x}'(t_0) \neq 0$